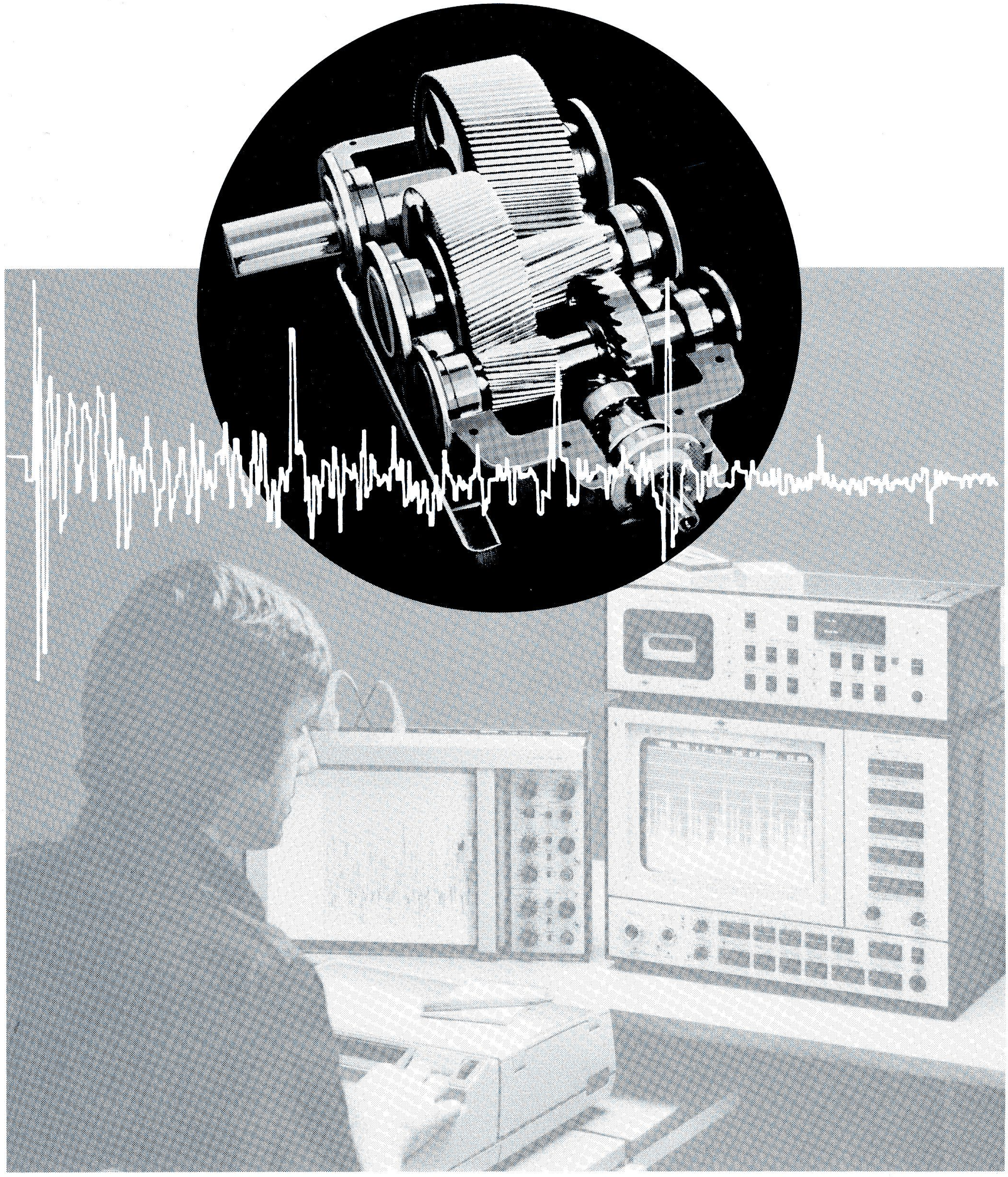
Application Notes

Cepstrum Analysis and Gearbox Fault Diagnosis



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Cepstrum Analysis and Gearbox Fault Diagnosis
Edition 2*

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1. Introduction

The cepstrum is defined in a number of different ways, but all can be considered as a spectrum of a logarithmic spectrum (i.e. logarithmic amplitude, but linear frequency scale). This means that cepstrum analysis can be used as a tool for the detection of periodicity in a spectrum, for example families of harmonics with uniform spacing. The logarithmic amplitude scale emphasizes the harmonic structure of the spectrum and reduces the influence of the somewhat random transmission path by which the signal travels from the source to the measurement point.

One of the earliest applications of the cepstrum was in speech analysis, with the aim of detecting the harmonic structure of voiced sounds and measuring voice pitch (Refs.1, 2). Another was in the study of signals containing echoes, which also give a periodic structure to the spectrum (Refs.1, 3).

The type of periodic structure considered here, however, is given by the families of sidebands commonly found in gearbox vibration spectra, and which often are an indication of faults of various kinds. Several people have shown that such sidebands can be considered to arise

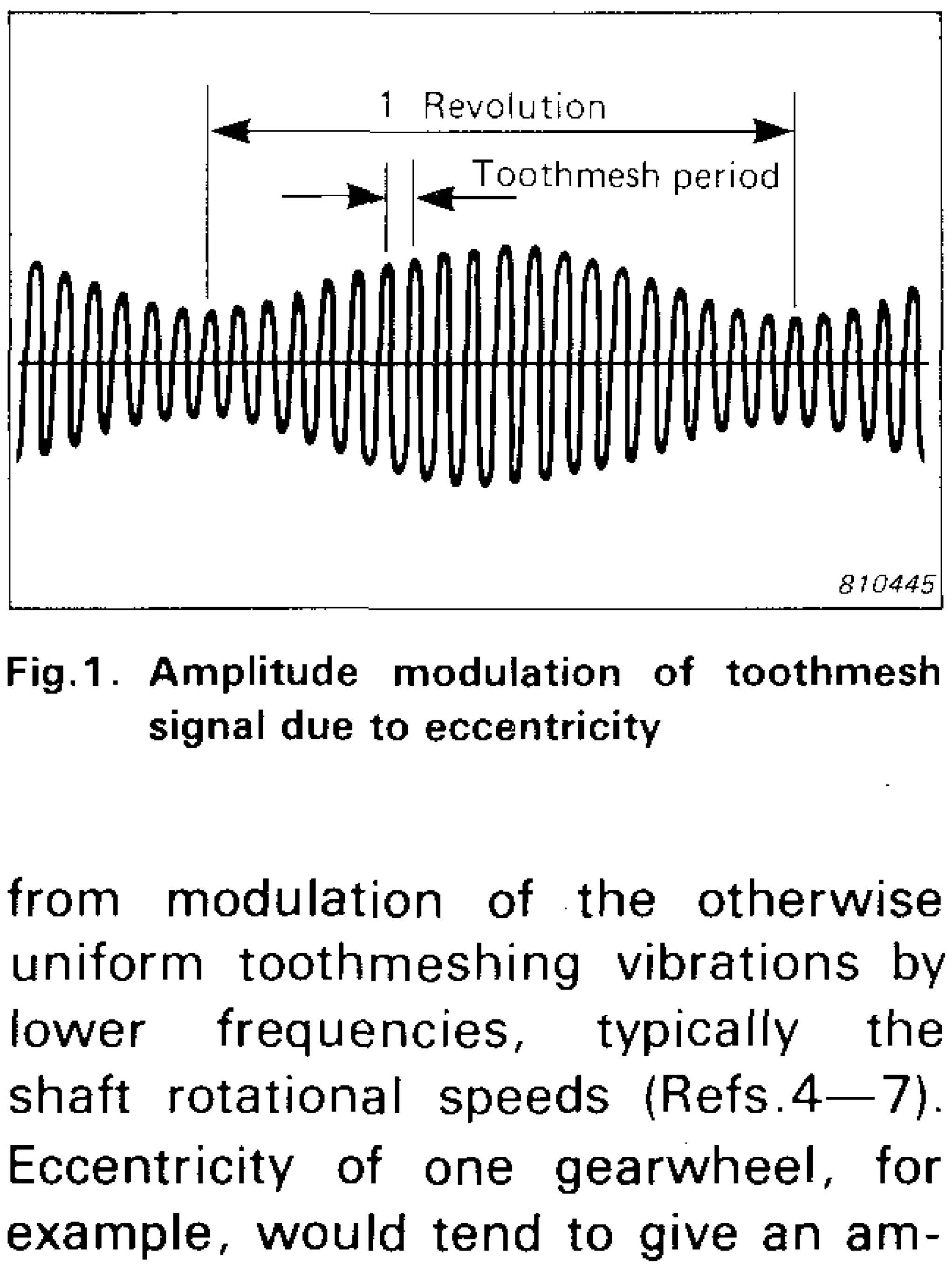

Fig.1. Amplitude modulation of toothmesh signal due to eccentricity

from modulation of the otherwise uniform toothmeshing vibrations by lower frequencies, typically the shaft rotational speeds (Refs.4—7). Eccentricity of one gearwheel, for example, would tend to give an amplitude modulation of the basic vibration generated at the toothmeshing frequency (and its harmonics) with an envelope period corresponding to the shaft rotation (Fig.1). At the same time, the variations in tooth contact pressure, which give the amplitude modulation, give rise to rotational speed fluctuations which cause frequency modulation at the same frequency. Thus, amplitude or frequency modulation seldom occurs in isolation, but on the other hand both give rise to a family of sidebands with the same spacing (the fundamental modulating frequency) and it is this sideband spacing which contains the basic diagnostic information as to the source of the modulation.

Appendix A contains a more detailed discussion of the generation of vibrations in gearboxes, and shows how uniformly distributed faults (i.e. the same for each tooth) only affect the toothmeshing harmonics, while more localised effects can be interpreted as combinations of amplitude and frequency modulation and additive impulses. The former effect shows up as sidebands around the toothmeshing frequency and its harmonics, while the latter tends to be limited to frequencies lower than the toothmeshing frequency.

Even for gearboxes in good condition, the spectra normally contain such sidebands but at a level which remains constant with time. Changes in the number and strength of the sidebands would generally indicate a deterioration in condition.

The spacing of these sidebands thus contains very valuable information as to the source of a vibration problem, often tracing it to a particular gear in a complicated gearbox, or in the case of load fluctuations (for example at the lobe-meshing frequency of a blower) establishing that the source of the modulation lies outside the gearbox itself. Often, however, as soon as there is more than one family of sidebands present at the one time, it can be

* Replaces Application Note No. 13—150 from 1973.

difficult to distinguish them by eye, and this is where the cepstrum is useful in separating the various periodicities. Here it can be considered as a diagnostic aid in interpreting the spectral information. The cepstrum may also be useful as a data reduction technique, effectively reducing a whole pattern of sidebands into a single line in the cepstrum, and easing the problem of monitoring changes in condition.

2. The Cepstrum

The cepstrum was originally defined (Ref.3) as the "power spectrum of the logarithm of the power spectrum", or mathematically:

$$C(\tau) = \left| \mathscr{F}\left\{ \log F_{xx}(f) \right\} \right|^2 \qquad (1)$$

where $F_{xx}(f)$ is the power spectrum of the time signal $f_x(t)$

$$\text{i.e. } F_{xx}(f) = \left| \mathscr{F}\left\{ f_x(t) \right\} \right|^2 \qquad (2)$$

and $\mathscr{F}\{\ \}$ represents the forward Fourier Transform of the bracketed quantity. Later, a newer definition was coined as the "inverse transform of the logarithm of the power spectrum", or

$$C(\tau) = \mathscr{F}^{-1}\left\{ \log F_{xx}(f) \right\} \qquad (3)$$

One of the reasons for using this definition is that it highlights the connection between the cepstrum and the autocorrelation function, which can be obtained as the inverse transform of the power spectrum, i.e.

$$R_{xx}(\tau) = \mathscr{F}^{-1}\left\{ F_{xx}(f) \right\} \qquad (4)$$

The definition (3) is also closer to that of the "complex cepstrum" (Ref.1) which is the "inverse transform of the complex logarithm of the complex spectrum" and in fact is identical to it for spectra with zero phase. The complex cepstrum is not further discussed here, however. Ref.1 contains an excellent discussion of the relationships between the various forms of the cepstrum, and many of their applications.

Note that the independent variable, τ, of the cepstrum has the dimensions of time, but is known as "quefrency". This is a useful terminology for those used to interpreting time signals in terms of their frequency content, since a "high quefrency" represents rapid fluctuations in the spectrum (small frequency spacings) and a "low quefrency" represents slow changes with frequency (large frequency spacings). Where peaks in the cepstrum result from families of sidebands, the quefrency of the peak represents the periodic time of the modulation, and its reciprocal the modulation frequency. Note that the quefrency says nothing about absolute frequency, only about frequency spacings.

In the application considered here, viz. the detection of peaks representing strong periodicity in the (logarithmic) spectrum, the choice of definition (1) or (3) is not so critical, as both will show distinct peaks in the same location. The squaring of Eqn.(1) tends to emphasize the largest peaks, which is not always an advantage, and in fact the author has often adopted a further definition, represented by the square root of Eqn.(1), viz. the amplitude spectrum rather than the power spectrum. This is in fact virtually the same as a rectified version (modulus) of Eqn.(3), since for a real, even function such as a power spectrum, the forward and inverse transforms give the same result except for a possible scaling factor.

The above discussion applies to the case where the 2-sided representation of the power spectrum (i.e. including both positive and negative frequencies) is used. Fourier theory (see, for example, Appendix B) shows that for such a real, even function the Fourier transform is also a real, even function and thus the cepstrum is represented by the real parts of the transform. If, on the other hand, the one-sided spectrum is transformed (i.e. the negative frequency components are set to zero) the real parts will still be effectively the same (and this thus provides an efficient way of achieving the same result) but the imaginary parts will represent the Hilbert transform of the real parts. The cepstrum obtained by squaring and adding the real and imaginary parts at each quefrency and extracting the square root will hereinafter be referred to as the "amplitude cepstrum (of the one-sided spectrum)". As will later be shown by an example, this is a very useful form when the spectrum being analyzed is obtained by a "zoom" process. Appendix B includes a discussion of the theoretical background.

2.1. The cepstrum vs. the autocorrelation function

In comparison with the autocorrelation function, the logarithms of the spectrum values are taken before inverse transformation (Eqns. (3) and (4)) and it is here that the main advantage of the cepstrum lies. Consider the normal case where the power spectrum of a (vibration) signal at an external measurement point is the product of the power spectrum of the source function and (the squared amplitude of) the frequency response function of the transmission path, i.e.

$$F_{yy}(f) = F_{xx}(f) \cdot \left| H_{xy}(f) \right|^2 \qquad (5)$$

The effect of taking the logarithm is to transform the multiplication to an addition, i.e.

$$\log F_{yy} = \log F_{xx} + 2 \log \left| H_{xy} \right| \qquad (6)$$

and the additive relationship is maintained by the linear Fourier transform, i.e.

$$\mathscr{F}^{-1}\{\log F_{yy}\} = \qquad (7)$$

$$\mathscr{F}^{-1}\{\log F_{xx}\} + \mathscr{F}^{-1}\{2 \log \left| H_{xy} \right|\}$$

meaning that source and transmission path effects are additive in the cepstrum. Since they often have quite different quefrency contents, they will then also be separated in the cepstrum, and under those conditions additive even using Eqn.(1), since cross terms vanish. By contrast, the autocorrelation obtained by inverse transformation of Eqn.(5) involves a convolution of the two effects, which is much more complicated.

3. Advantages of Cepstrum vs. Spectrum Analysis

The cepstrum can be considered as an aid to the interpretation of the spectrum, in particular with respect to sideband families, because it presents the information in a more efficient manner. One advantage resulting directly from the considerations of the previous paragraph is the lack of sensitivity to transmission path effects. Small changes in positioning of an accelerometer, for example, can modify the overall shape of the spectrum and thus influence the level of individual sidebands. The cepstrum component corresponding to a given sideband spacing, however, is an average sideband height over the whole spectrum and is much less likely to be affected. Fig.2 illustrates a typical case with spectra taken from two separate measurement points on the same gearbox, but representing the same internal condition. The spectra are quite different in shape (for example at 2,6 kHz there is a peak in one spectrum and a trough in the other) but the significant cepstrum components are almost identical. The different spectrum shapes show up in the "low quefrency" range of the cepstra.

Another effect which can modify the shape of the spectrum, even with the same degree of fault in the gearbox, are the phase relationships of amplitude and frequency modulation at the same frequency. Even though they are coupled at the source, the frequency modulation is most directly related to the torsional properties of the system, while the amplitude modulation is more affected by the lateral response properties, and this can explain phase shifts. Either amplitude or frequency modulation in isolation tends to give symmetrical families of sidebands around the carrier frequency, but the combination will usually give reinforcement on one side and cancellation on the other, which thus partly explains the non-

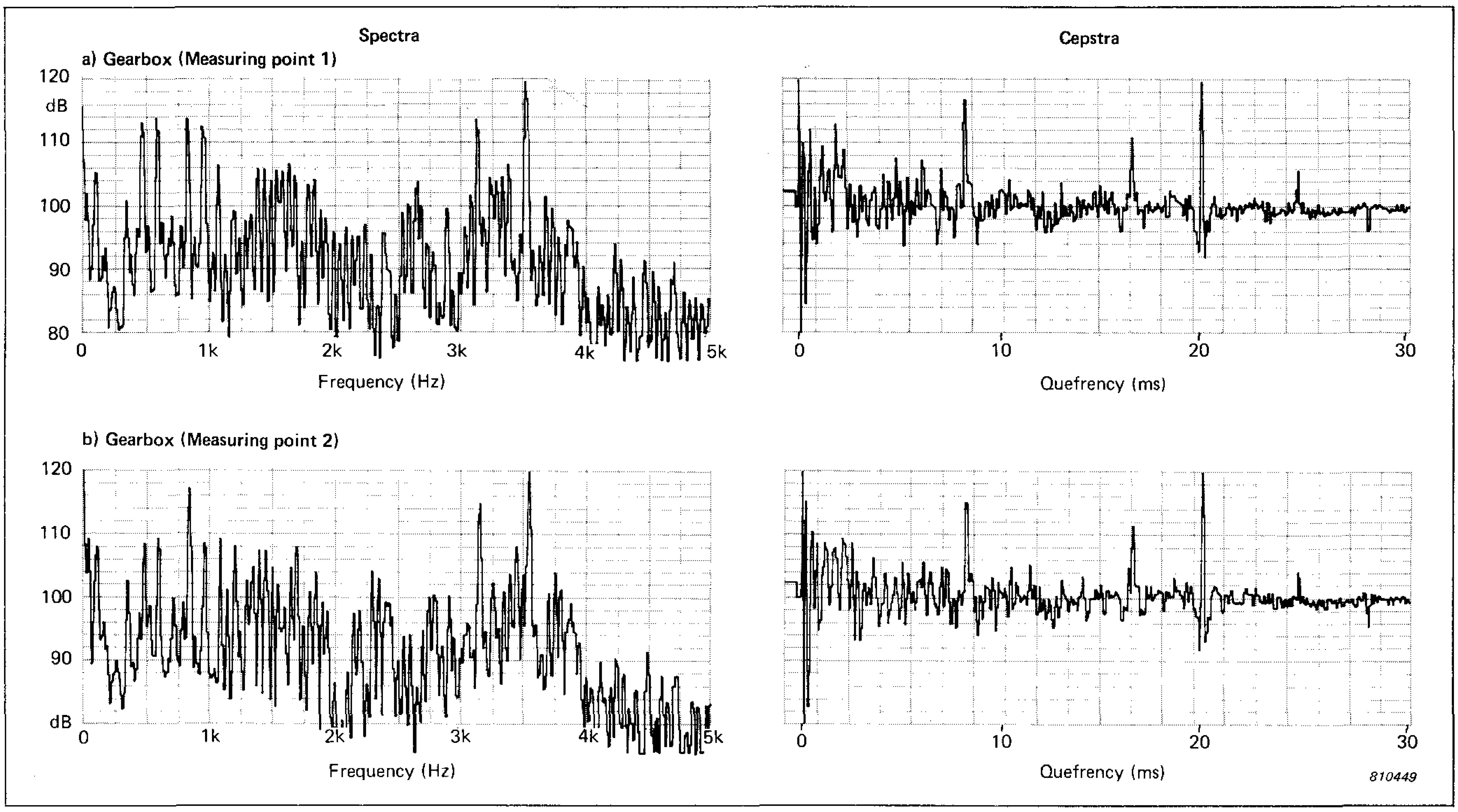

Fig.2. Lack of sensitivity of cepstrum to transmission path effects

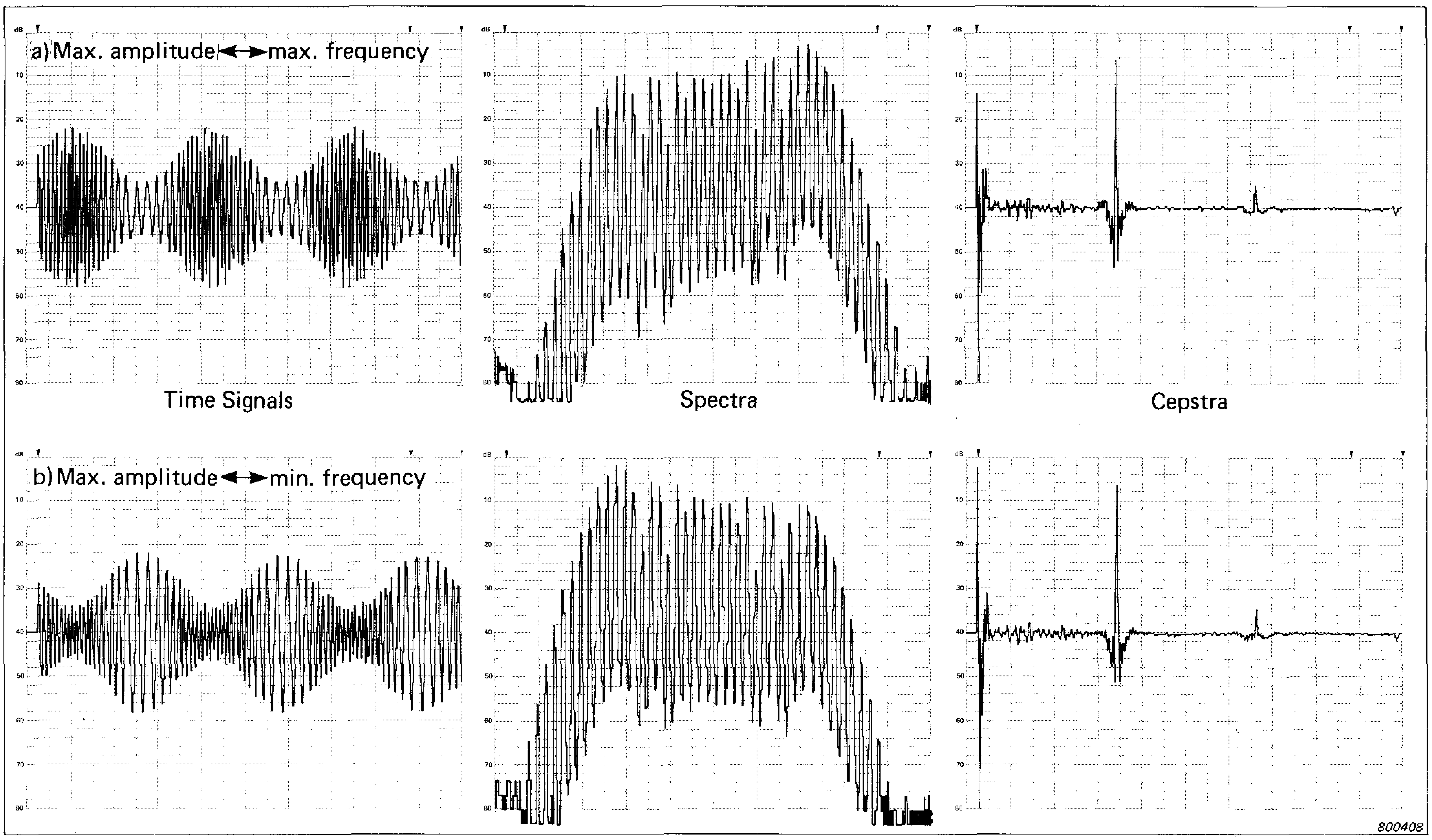

Fig.3. Lack of sensitivity of cepstrum to phase relationships of amplitude and frequency modulation

symmetry commonly found in practice. Transmission path effects also modify the symmetry, of course.

Fig.3 is a numerically generated example showing the lack of sensitivity of the cepstrum to the phase relationships of amplitude and frequency modulation. A phase shift of 180° gives a difference in individual sidebands of up to 8 dB, depending on whether the high or low frequency part of the signal is emphasized by the amplitude modulation, but the cepstrum components corresponding to the sideband spacing remain unchanged. Other intermediate phase relationships also gave the same result.

The two advantages cited above have to do with detection of significant changes in condition, and show that the cepstrum is less sensitive to secondary effects than the level of individual sidebands, since it gives a measure of average sideband "activity" over the whole spectrum. An advantage also results from the fact that the whole family of sidebands is reduced basically to one line in the cepstrum, which of course is considerably simpler to monitor. This statement requires some further explanation because cepstra in practice usually contain several "rahmonics" (harmonics in the cepstrum) corresponding to each sideband spacing. See for example Fig.2, where the first three rahmonics of the 8,28 ms component are present while only the first rahmonic of the 20,1 ms component is within the display range. However, it will be appreciated that the first or fundamental rahmonic contains the significant information on the average sideband height, while the others represent the distortion and are thus influenced by artefacts such as the filter characteristic and its interaction with the sideband spacing. Thus, having recognized a series of rahmonics, it is generally sufficient to consider the first and neglect the higher order rahmonics. An example will later be given, however, where a higher rahmonic represents a component in its own right, but this is apparent from the way in which the rahmonics fall off with quefrency.

The cepstrum also gives advantages with respect to diagnosis (as opposed to detection) of faults. These have to do with the ability of the cepstrum to detect periodicity in the spectrum which is not immediately apparent to the eye, and the accuracy with which this can be measured.

Fig.4 illustrates the first point. Fig.4(a) is a 400-line baseband spectrum from 0 — 20 kHz of a gearbox vibration signal containing at least the first three harmonics of the toothmeshing frequency (4,3 kHz). In this analysis the sidebands are not resolved. Fig.4(b) is a 2000-line composite zoom spectrum covering the range 3,5 — 13,5 kHz, and thus including the first three toothmeshing harmonics but excluding the low harmonics of both shaft speeds. This range was selected from the 4000 lines possible (from a single 10 K time record) using the High Resolution Signal Analyzer Type 2033, and was in fact obtained from exactly the same time record as the "Scan Average" of Fig.4(a). To make clearer the resolution achieved with this zoom analysis, Fig.4(c) shows the 400-line section extending from 7,500 — 9,500 Hz (and thus including the second toothmesh harmonic) expanded in the X-direction. Exactly the same data are reproduced as in the equivalent section of Fig.4(b).

The 2000-line spectrum of Fig.4(b) was read back into the anal-

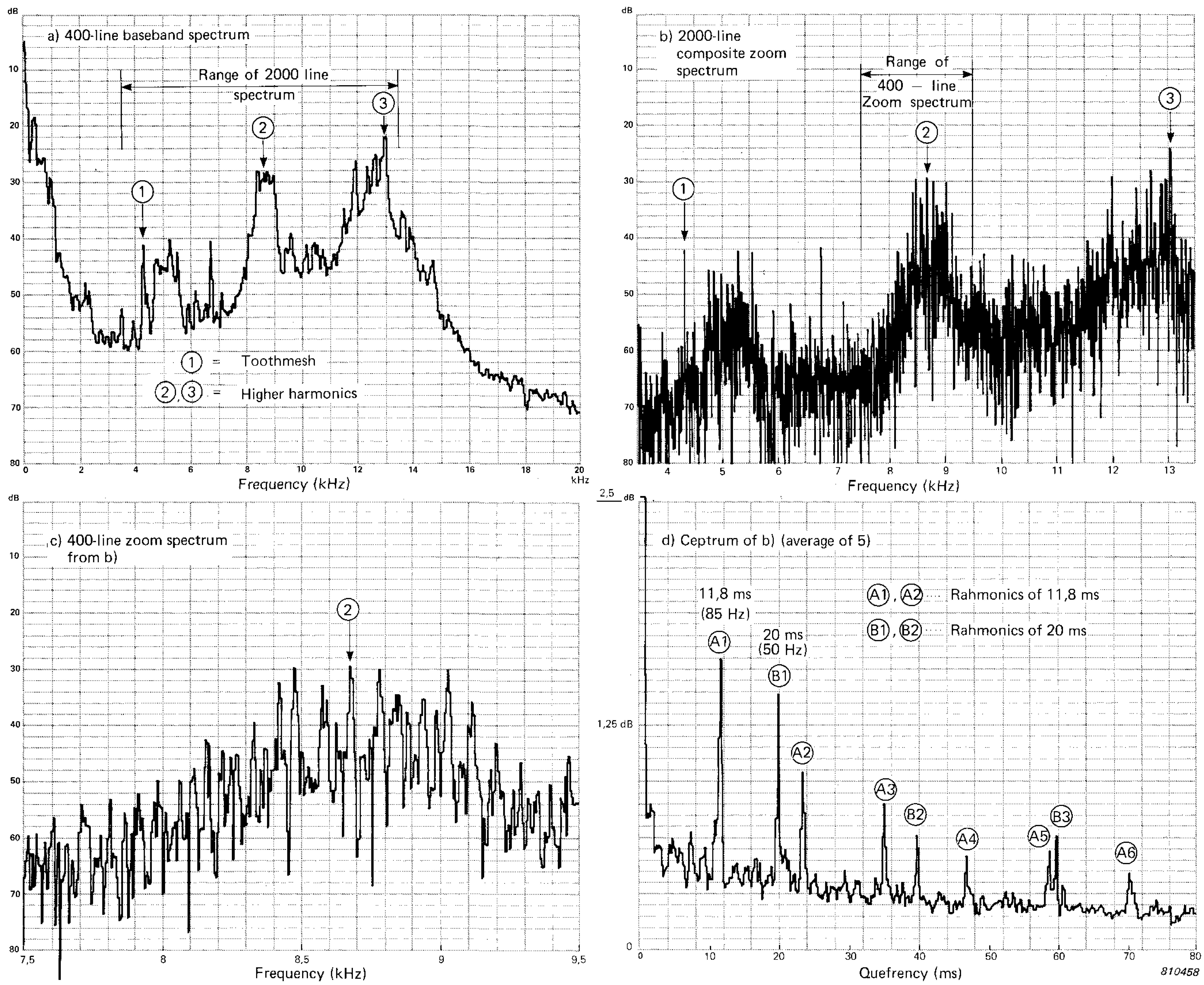

Fig.4. **(a) 400-line baseband spectrum**
(b) 2000-line composite zoom spectrum
(c) 400-line zoom spectrum from b)
(d) Cepstrum of 2000-line spectrum b) (average of 5)

yzer digitally as a time signal (using an HP 9825 calculator) and then a "scan" analysis performed to obtain the cepstrum corresponding to the entire spectrum. (If it had contained significant information, the entire 4000-line spectrum could have been read back into the 10 K memory of the Analyzer Type 2033). This thus represents the "amplitude cepstrum of the one-sided spectrum", and is reproduced as Fig.4(d) (actually Fig.4(d) is the average of 5 such cepstra). The cepstrum shows that only two components, corresponding to the two shaft speeds 85 Hz *and* 50 Hz, *are important* over the analyzed range of the spectrum. All significant cepstrum components are rahmonics of one of the two shaft speeds. This periodicity is not so apparent to the eye in the zoom spectra, since the mixture of the two periodicities gives a "quasi-periodic" structure.

The second diagnostic advantage is illustrated by Fig.5. This shows spectra and cepstra for two truck gearboxes, in good and bad condition respectively, running on a test stand with first gear engaged. The good gearbox shows no marked spectrum periodicity, but the spectrum of the bad one contains a *large number of sidebands with a* spacing of approx. 10 Hz. It is difficult to determine the spacing much more accurately in the spectrum. In the cepstrum, the corresponding quefrency is 95,9 ms (10,4 Hz) while there is also a series of rahmonics corresponding to the input shaft speed (28,1 ms — 35,6 Hz). With that input speed, the output shaft speed would have been 5,4 Hz, and it was first suspected that the modulating frequency was its second harmonic. However, that would have been 10,8 Hz, not 10,4 Hz, and in the end it was found that the latter corresponded exactly to second gear speed, indicating that this was the modulation *source, even though first gear was* engaged, and second gear was unloaded.

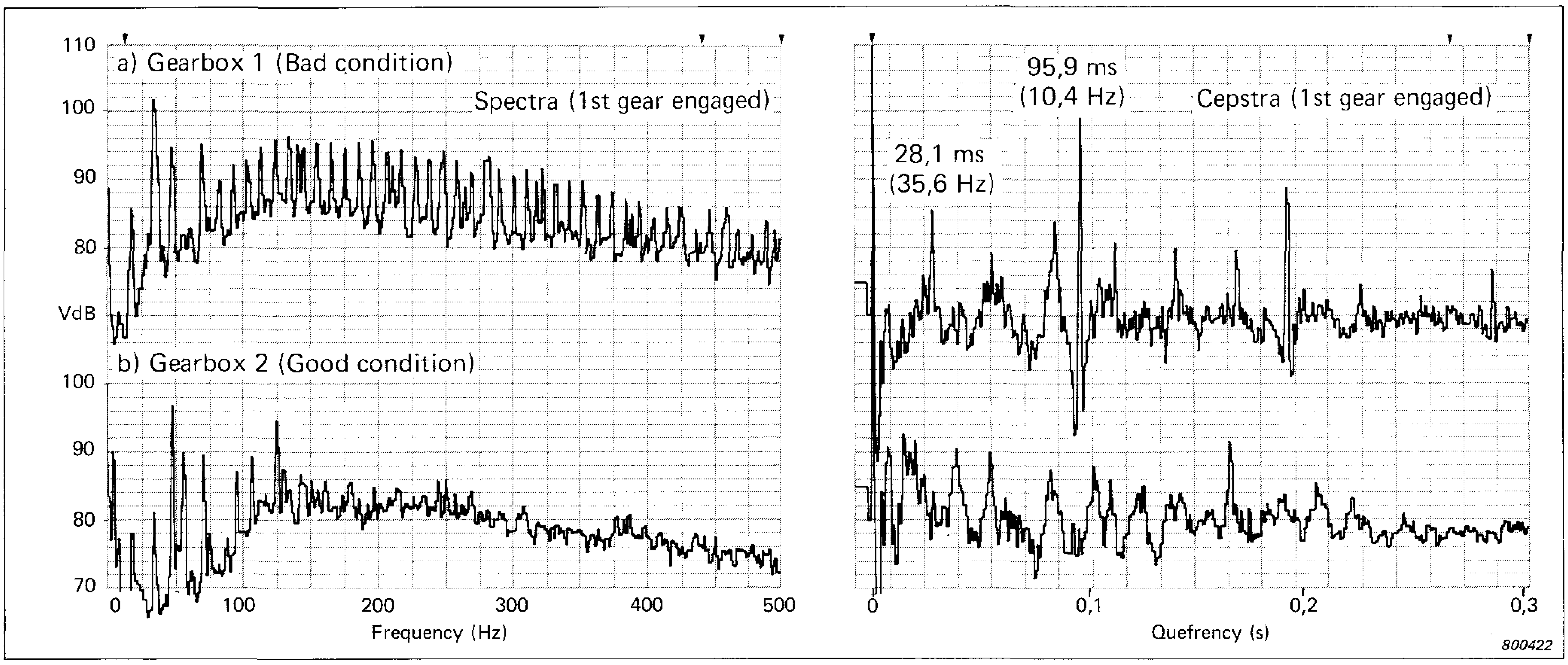

Fig.5. Spectra and cepstra from truck gearboxes in good and bad condition

4. Practical Considerations

In theory, problems arise in *spectra consisting of discrete frequency components* (such as gearbox vibration spectra) because the logarithm does not exist for the zeroes between the components. However, in practice there would normally be a base noise level in each spectrum, or in any case a lower limit determined by the dynamic range of the system. Fig.6 illustrates how the signal-to-noise ratio in a spectrum would have a direct effect on the cepstrum component corresponding to a series of periodic spectrum components, and for this reason it is only valid to compare cepstra obtained with similar base noise conditions.

It should be noted in this connection that this is only partly determined by the signal, but also influenced by the choice of analyzer *bandwidth, since each halving of* the bandwidth would lift discrete frequency components a further 3 dB out of the noise. On the other hand, this halving of the bandwidth (for the same component spacing) would tend to reduce the first rahmonic with respect to the others so that the overall effect is quite complex.

For condition monitoring applications it may be advantageous to artificially limit the "dynamic range" of the analysis so as to detect a bigger change in the cepstrum from either or both of the following two effects:

(a) An increase in the toothmeshing harmonics carrying sidebands up with them.
(b) An increase in the number and/or strength of sidebands *with respect to constant tooth*meshing harmonics.

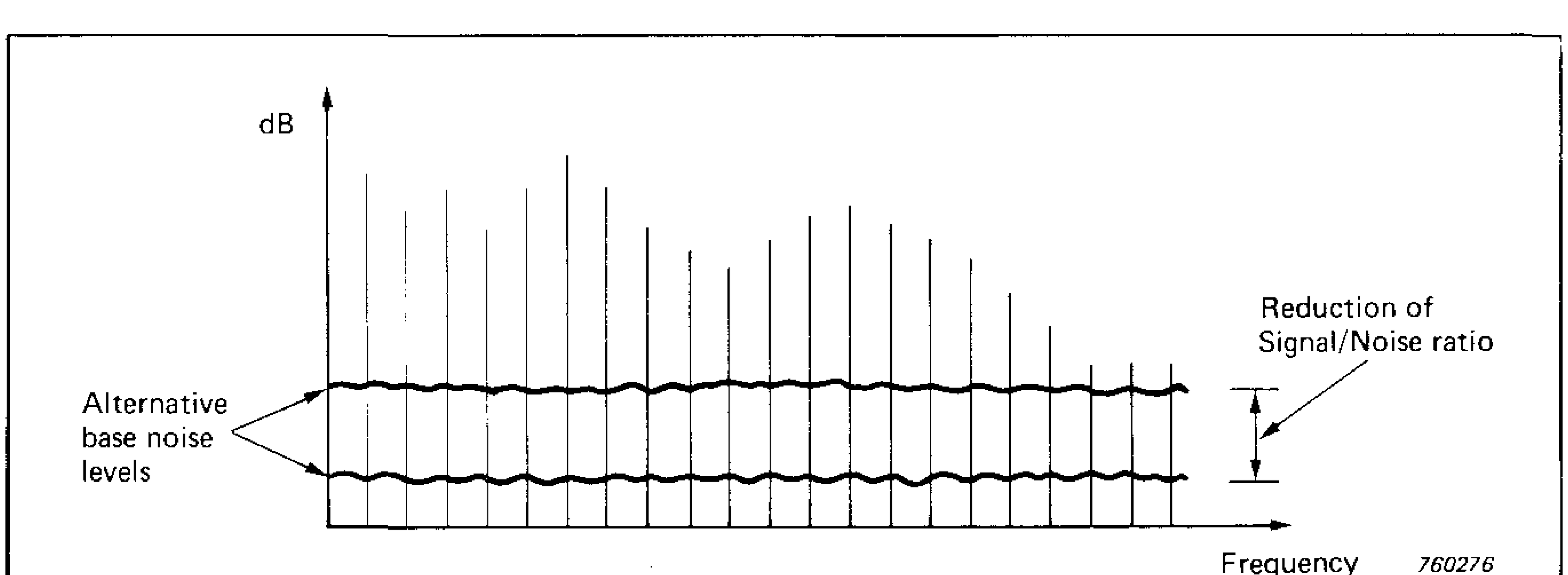

Fig.6. Effect of noise level on size of cepstrum component

Both of these effects would signify deterioration of one kind or another, although in order to determine which kind it would probably be necessary to take the spectrum changes into account as well, as discussed in Appendix A.

Another practical point which could affect cepstrum values quite considerably is the choice of vibration parameter, i.e. acceleration or velocity. In theory, the choice of parameter should not matter very much, since this only alters the local slope of the spectrum, basically a low quefrency effect. However, as illustrated in Fig.7, it would generally be best to choose that parameter *which has the flattest spectrum*, so that significant components do not fall outside the dynamic range.

The final point having to do with artificial limitation of significant cepstrum components is the question of "bridging" between adjacent components. As illustrated in Fig.8, this can occur if:

(a) The filter shape factor is too poor for a given bandwidth.
(b) The spacing between the com-

ponents is too small with respect to the bandwidth.

The method recommended here for calculation of the cepstrum involves an FFT analyzer connected to a desktop calculator (see Appendix B) and thus the filter characteristic is determined by the time-weighting function used for the FFT analysis. Since gearbox vibration signals are continuous it will normally be desirable to use Hanning weighting in the initial spectrum analysis, this giving a relatively good filter characteristic. It will be found that, with Hanning weighting, the separation of adjacent discrete frequency components must be at least 8 lines to ensure that bridging is suppressed to below —50 dB, and this is thus recommended as the minimum spacing to avoid this problem (unless the components in any case do not protrude so much from the background noise).

This means that in a normal 400-line FFT analysis it will only be possible to include about 50 components at the minimum spacing and thus explains why it would often be necessary to employ "zoom" analysis to achieve sufficient resolution. For example, for the hypothetical 20-tooth gear of Fig.9, these 50 components (in a baseband spectrum) would only extend up to between the second and third toothmeshing harmonics. Even restricting the analysis to the first toothmeshing harmonic would only allow for gears with up to about 40 teeth. On the other hand, a zoom factor of 10, giving 10 times better resolution, would make it possible to analyze gears with up to about 450 teeth (or 150 if it is desired to include up to the third toothmeshing harmonic).

The above reasoning means that it would often be necessary to perform a "zoom" analysis in order to obtain sufficient resolution in the original spectrum, before performing the cepstrum analysis. It is thus relevant to consider the effects of performing a cepstrum analysis on such a zoomed spectrum.

As pointed out in Appendix B, the cepstrum defined according to Eqn (3) will give positive peaks when the sidebands correspond to harmonic frequencies (i.e. the family of sidebands when projected downwards includes zero frequency, see Fig.B4a). If the projection of a sideband family does not pass through zero frequency this will not be the case, and in particular if they are evenly spaced around zero (e.g. a 2-sided spectrum of odd harmonics, see Fig.B4b) then the result in the cepstrum will be an alternating series of rahmonics with the first one negative.

In particular, when the cepstrum is carried out on a zoomed spectrum, the lower limiting frequency is interpreted as being zero, and the

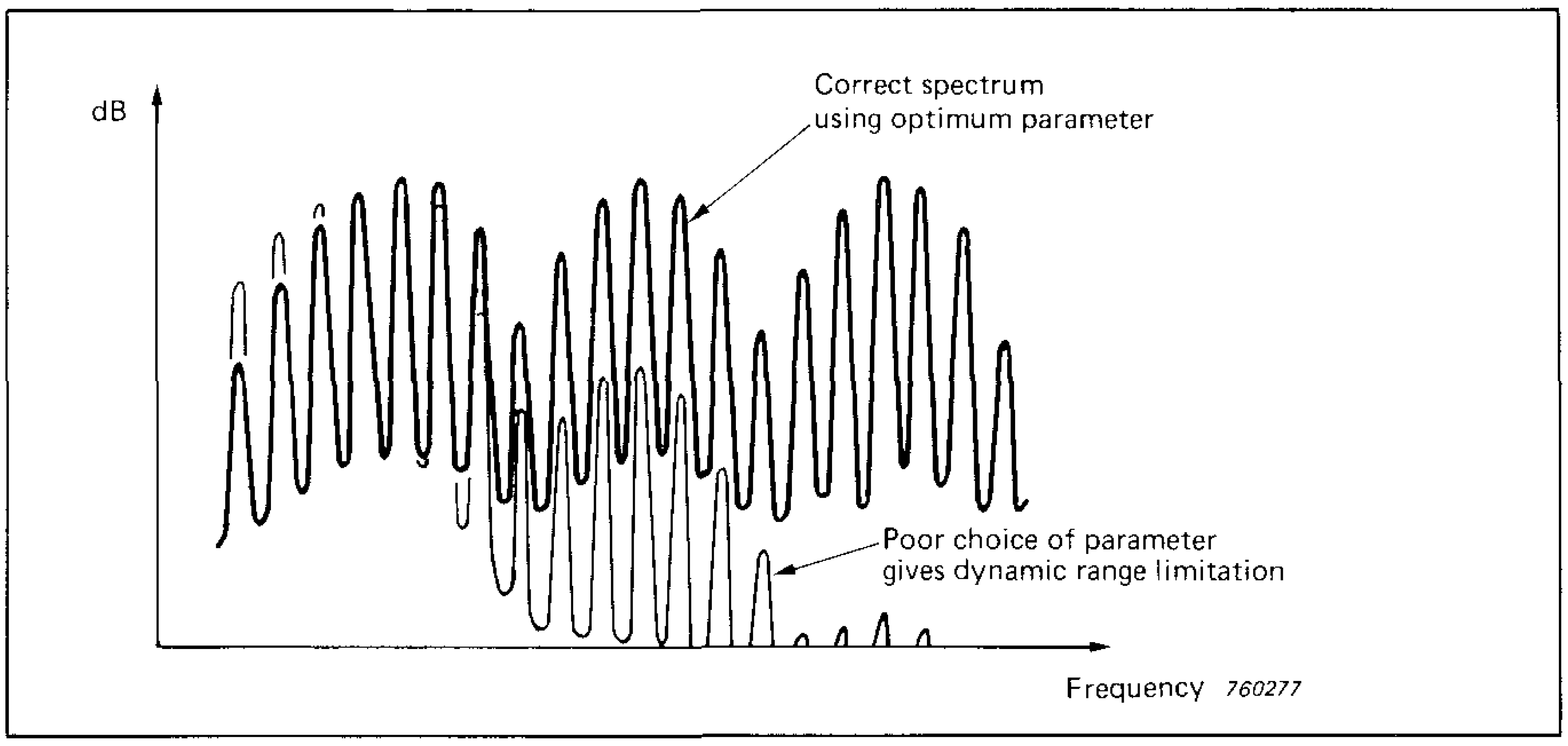

Fig.7. Choice of correct parameter for cepstrum analysis

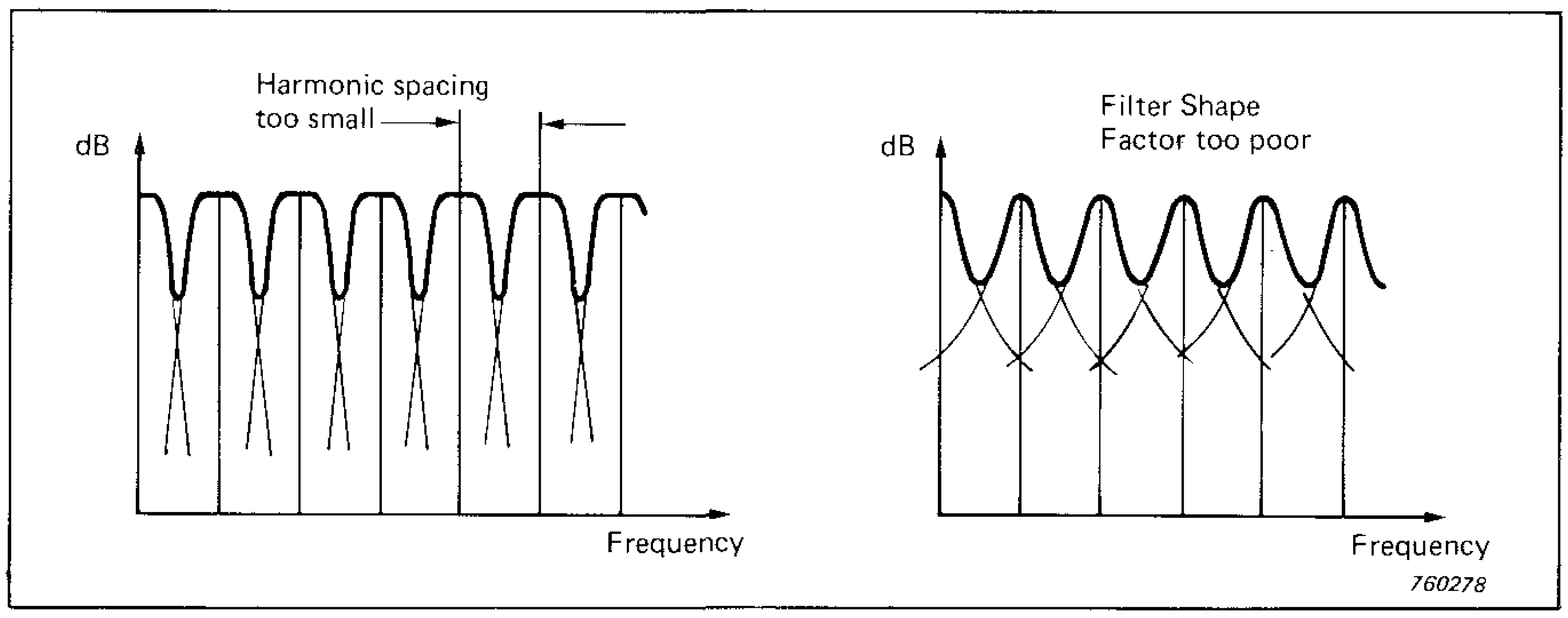

Fig.8. Effect of filter bandwidth and shape factor

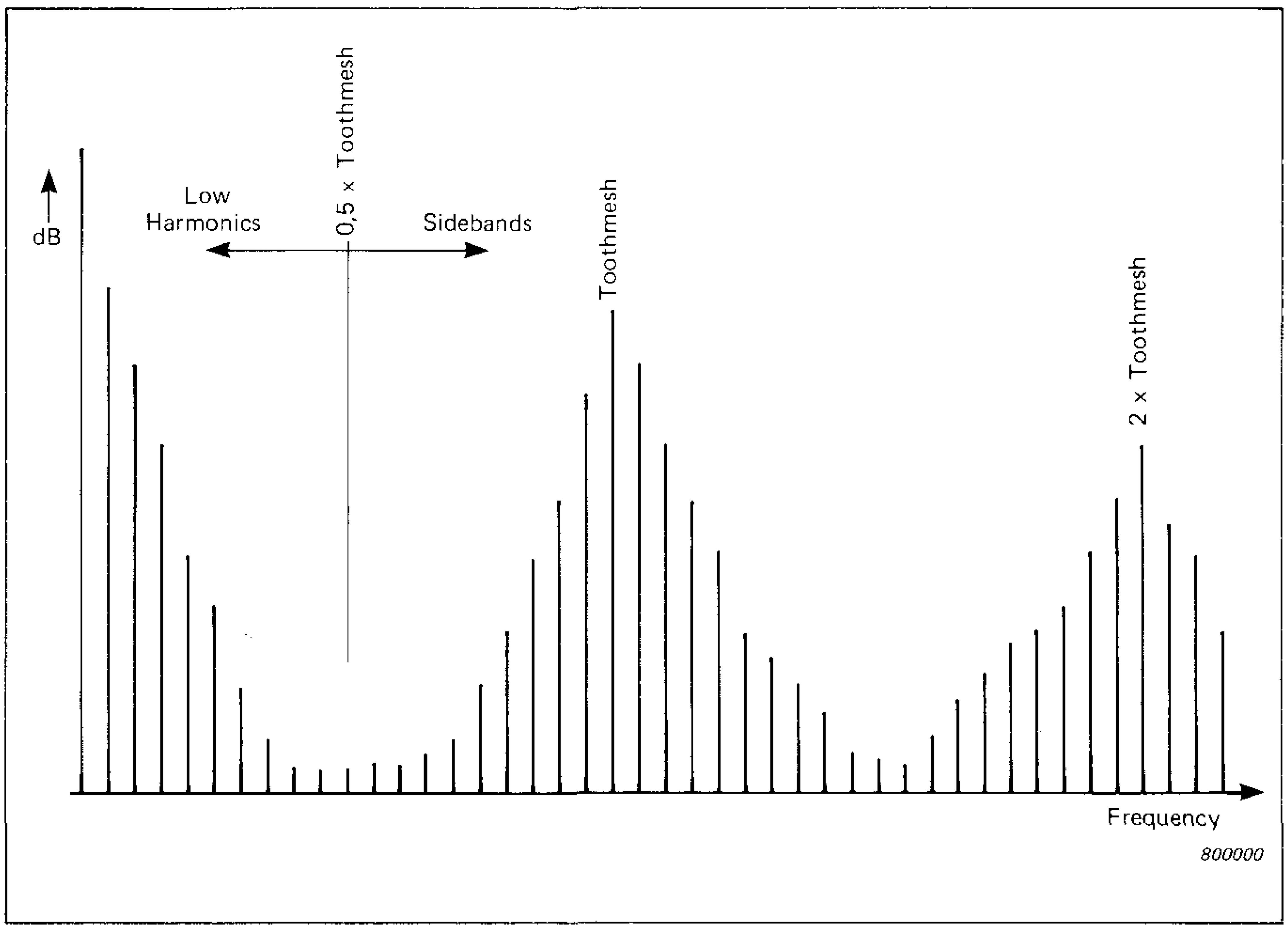

Fig.9. Hypothetical spectrum from a 20-tooth gear

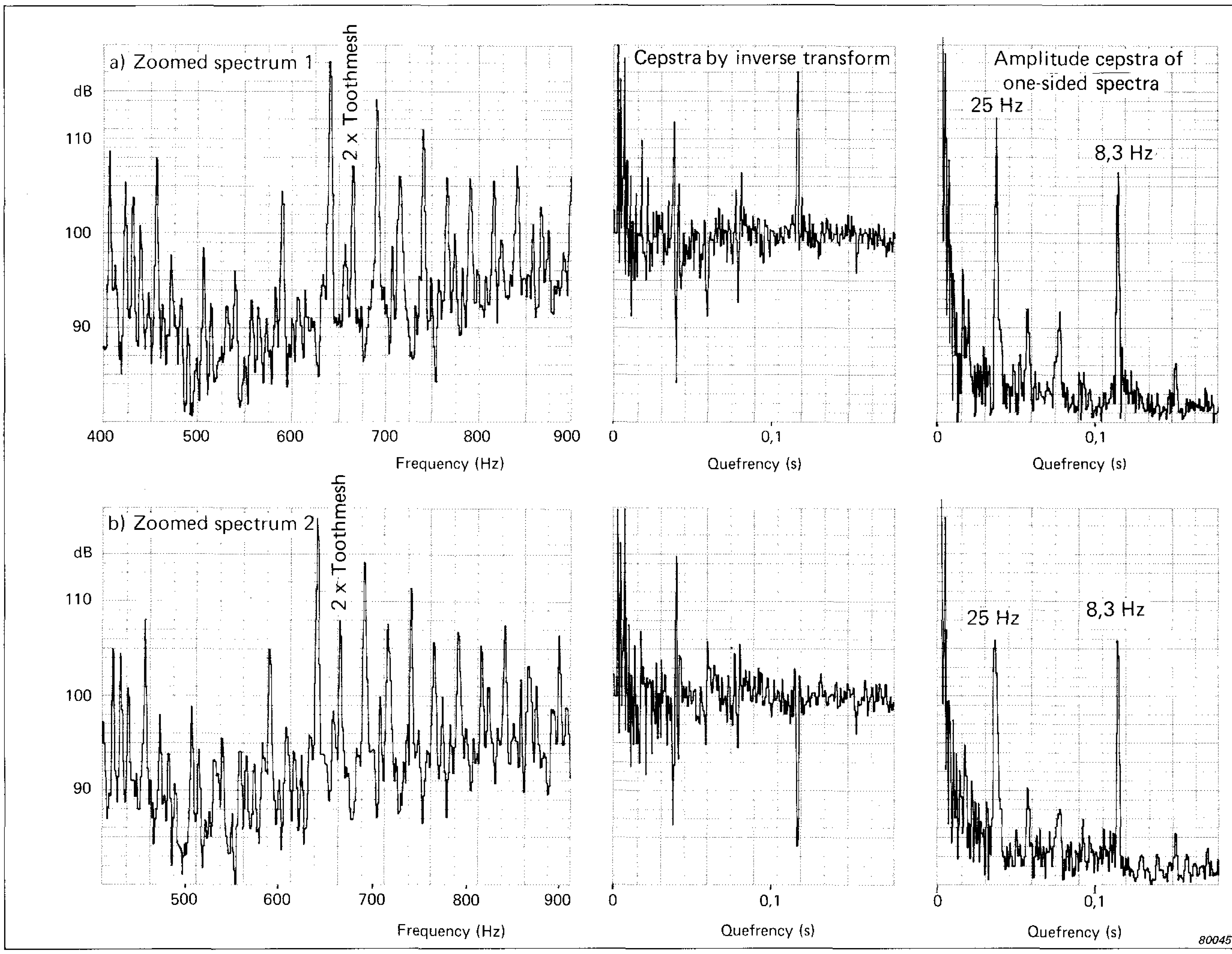

Fig.10. Cepstra of slightly displaced zoom spectra

significance of "phasing" is lost. Fig.10 shows a typical case of two zoomed spectra centred approximately on the second toothmeshing harmonic (666 Hz) of a 40-tooth pinion rotating at 8,3 Hz, but with a 10-line displacement between the 400-line zoomed spectra. The sign of the equivalent components changes completely in the inverse transform cepstra (Eqn.(3)) whereas the "amplitude cepstra of the one-sided spectra" are similar and less confusing. This is thus one application where the latter definition is to be recommended.

On the other hand, performing a cepstrum analysis on a single zoomed spectrum does tend to overemphasize that particular part of the spectrum (see for example the differences between the cepstra of Fig.10(a) and (b) resulting from such a small displacement of the spectrum). It is to be expected that a cepstrum over a wider frequency range would give a more representative result, and it is here that the method described in connection with Fig.4 is useful, i.e., the ability to perform a cepstrum analysis of a spectrum of up to 4000 lines. This possibility is unique to the B & K Analyzer Type 2033 because of its unique method of zoom (Ref.8).

Fig.11 shows the results of this type of analysis (on a 1200-line spectrum extending from 150 — 1650 Hz) for the same gearbox as in Fig.10, both before and after repair. Note that this is the same case as discussed in connection with Fig.A7 of Appendix A. What is of interest here is that two main cepstrum components dominate, even over this range including the first 4 toothmeshing harmonics (but eliminating frequencies below approx. half the toothmeshing frequency). Both the 40 ms and 120 ms quefrency components are significant before repair, indicating that there is modulation both at the shaft speed (8,3 Hz) and its third harmonic (25 Hz). This is the case referred to earlier where the third rahmonic (120 ms) is obviously a fundamental component in its own right, which can be seen by the way in which the rahmonics fall off with quefrency. The shaft speed modulation was probably due to misalignment which was corrected during repair, while the third harmonic was found to be due to a measurable "triangularity" of the gear. After repair the 40 ms component has disappeared completely and the shaft speed component has fallen drastically. It was found that the triangularity was due to excessive lapping during manufacture and the repair consisted in part in reversing the pinion on its shaft, utilising the unused flanks. Fig.11(c) shows that 4 years later the same problem has not developed again although there is some indication of wear in the development of the toothmesh harmonics.

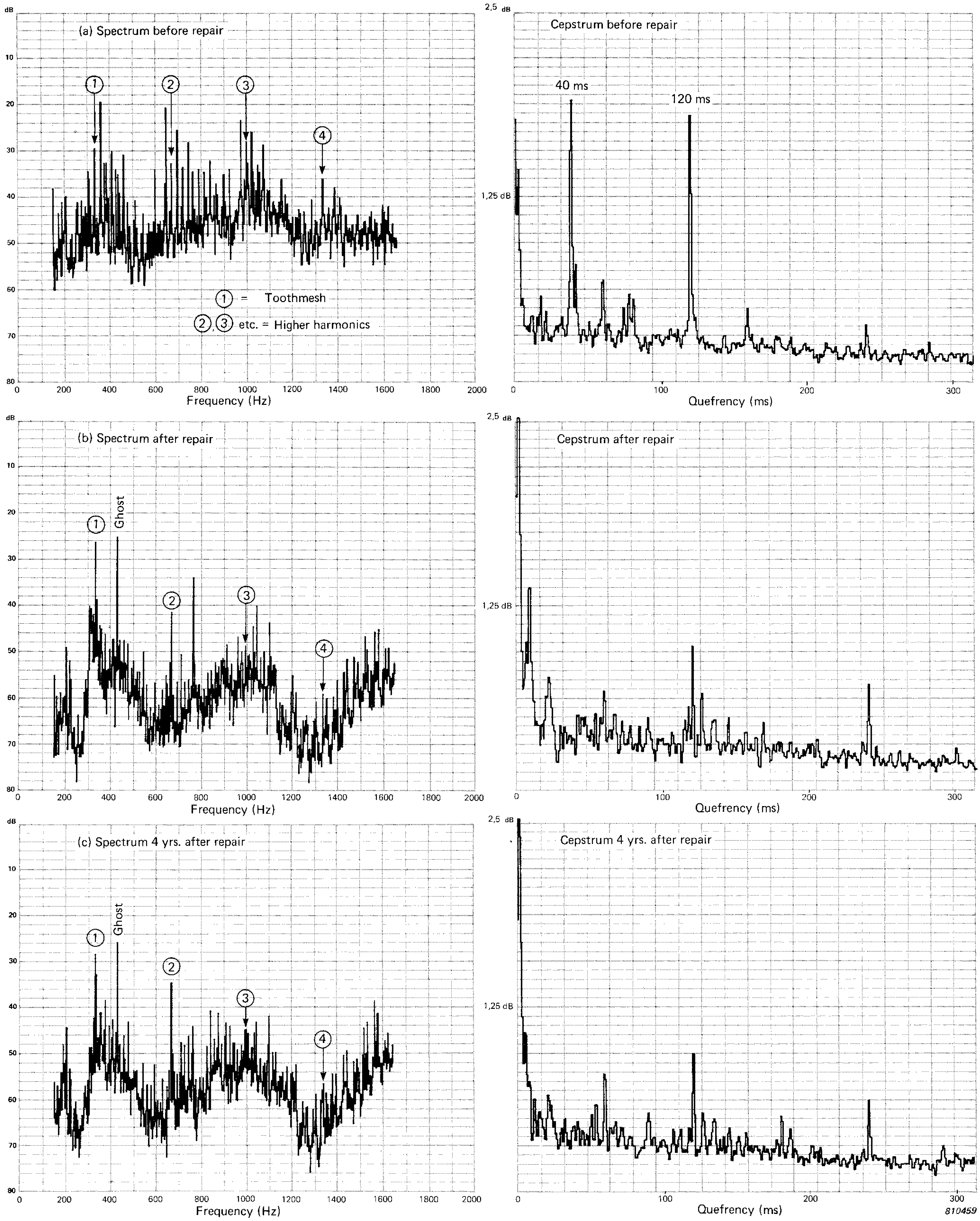

Fig.11. Spectra and cepstra for a gearbox
(a) Before repair
(b) Immediately after repair
(c) 4 years after repair

Conclusions

(1) Gearbox vibration spectra normally contain sidebands due to modulation of toothmeshing frequencies and their harmonics, and increases in the number and/or strength of such sidebands usually indicate deteriorating condition.

(2) The spacing of such sidebands gives valuable diagnostic information as to their source, since both amplitude and frequency modulation at the same frequency give sidebands with the same spacing. Most faults give a combination of amplitude and frequency modulation at the same time, the relative proportions and phase relationships being dependent in a complex way on the response properties of the individual machine, and so a division into the two categories is less useful than a measure of the overall sideband "activity" with a given spacing.

(3) The cepstrum, being a spectrum of a logarithmic spectrum, is admirably suited both for detecting the presence and/or growth of sidebands in gearbox vibration spectra, and for indicating their mean spacing over the entire spectrum, and is thus applicable to both detection and diagnosis of faults.

(4) With respect to fault detection, the cepstrum has advantages (in comparison with normal spectrum analysis) in being able to extract spectrum periodicity not immediately apparent to the eye, and in being insensitive to secondary effects such as signal transmission path and phase relationships of amplitude and frequency modulation.

(5) With respect to fault diagnosis, the cepstrum has advantages in measuring the average sideband spacing over a very wide range of the spectrum, thus allowing a very accurate measure of the spacing, and being representative of the whole spectrum. With normal spectrum analysis, one can either "not see the woods for trees" (zoom analysis) or "not see the trees for woods" (baseband analysis). The cepstrum has the ability to concentrate the significant sideband information in a very efficient manner. Often, both cepstral and spectral information would be useful in making a diagnosis, since, as discussed in Appendix A, changes in the shape of the spectrum, to which the cepstrum is not sensitive, can be important diagnostically. As an example, the question of whether a fault is localised, distributed, or the same for each tooth will most easily be seen in the spectrum.

(6) The cepstrum is more sensitive than normal spectrum analysis to certain artefacts resulting from the choice of spectrum parameters, and calculation method, and the discussion here should be useful as a guide to this choice.

(7) It is found that some of the major benefits of cepstrum analysis arise from its ability to analyze spectra with a very fine resolution, and thus the ability of the B & K High Resolution Analyzer Type 2033 to produce and analyze 4000-line spectra is very valuable in this respect. It can virtually only be matched by computer systems with large transform sizes, and the combination of the 2033 FFT analyzer with a desk-top calculator would usually be considerably less expensive, and faster, than such systems. At the same time, the combination of analyzer and calculator is more portable, virtually as flexible, and automatically includes necessary accessories such as antialiasing filters, A/D converter, digital display and output to a graphic recorder.

The Analyzer Type 2031 has resolution limited to 400 lines, and is thus limited to gears with smaller numbers of teeth (< 50).

B & K's Machine Health Monitoring Program No. BZ14 for use in the HP 9825 calculator includes a cepstrum program for use with either Analyzer Type 2031 or 2033. It is limited to 400-line spectra, but in the case of Analyzer Type 2033 the 400-line spectrum can be obtained by the "zoom" process. Appendix B contains a listing of the somewhat simpler program used to generate the "amplitude cepstrum" of spectra with up to 4000-lines (c.f. Fig.4(d)). Appendix B also contains some further details of the use of both programs.

Appendix A — Gearbox Vibrations

Even though the vibration spectra produced by gearboxes often appear quite complicated, they can usually be broken down into a combination of the following effects:

1) **Harmonics of the toothmeshing frequency** — representing those deviations from the ideal tooth profile which are the same for each toothmesh.

2) **Ghost components** — which appear like toothmeshing components but corresponding to a different number of teeth to those actually cut. They can be

traced back to the number of teeth on the index wheel of the gearcutting machine, and are due to errors in these teeth (and its mating gear).

3) **Sidebands** — due to modulation of the otherwise uniform toothmeshing signal, and either representing slow changes (e.g. eccentricity) or sudden variations due to local faults.

4) **Low harmonics of the shaft speed** — due to additive impulses repeated once per revolution.

5) **Intermodulation components** — representing sum and difference frequencies of the other components, in particular when the latter are close to each other.

Each of these effects will be discussed in more detail with examples.

Toothmeshing Harmonics

The deviations from the ideal profile which are the same for each toothmesh and which therefore give a signal periodic at the toothmeshing rate can be ascribed to two main sources. On the one hand there is the tooth deflection under load which varies as the load is shared between different numbers of teeth during each mesh cycle, and on the other hand there are deviations which result from uniform wear.

Because the tooth deformation component is so load dependent, it means that to obtain repetitive spectra it is essential to make measurements always with the same loading. The load must also be sufficient to ensure that the teeth are permanently in contact, since otherwise not only would a source of randomness be introduced, but the gears would also fail more rapidly (Ref.9).

With constant load, however, any changes in the toothmeshing frequency and its harmonics would most likely be due to wear, or at least that part of it which is the same for all teeth. Fig.A1 illustrates a typical wear profile as predicted by a wear equation developed in Ref.9. Wear is greatest on either side of the pitch circle because of a sliding action there, whereas at the pitch circle there is a rolling action only. It will be seen that this profile error would tend to give a considerable distortion of the toothmeshing frequency, and thus wear is often more evident at the higher harmonics of toothmeshing than at the toothmeshing frequency itself (Fig.A2). Usually it is advisable to monitor at least the first three toothmesh harmonics to detect tooth wear.

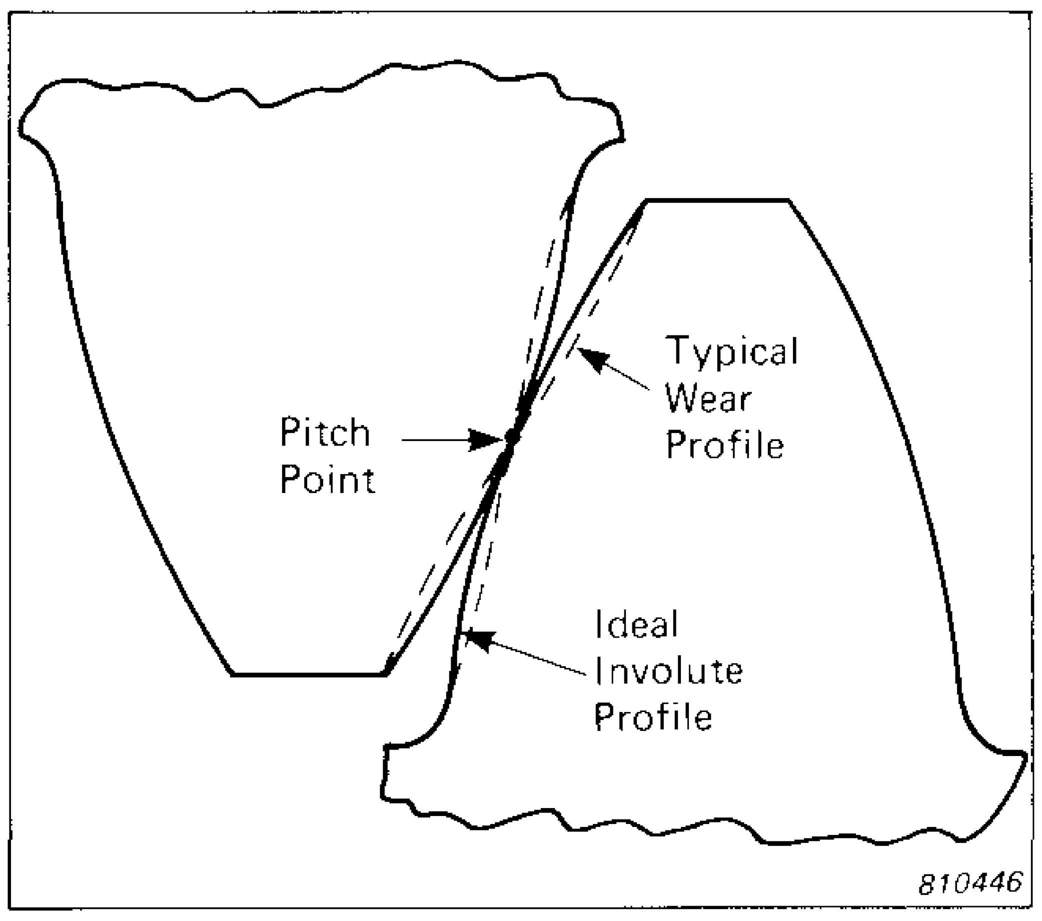

Fig.A1. Typical wear profile for a gear

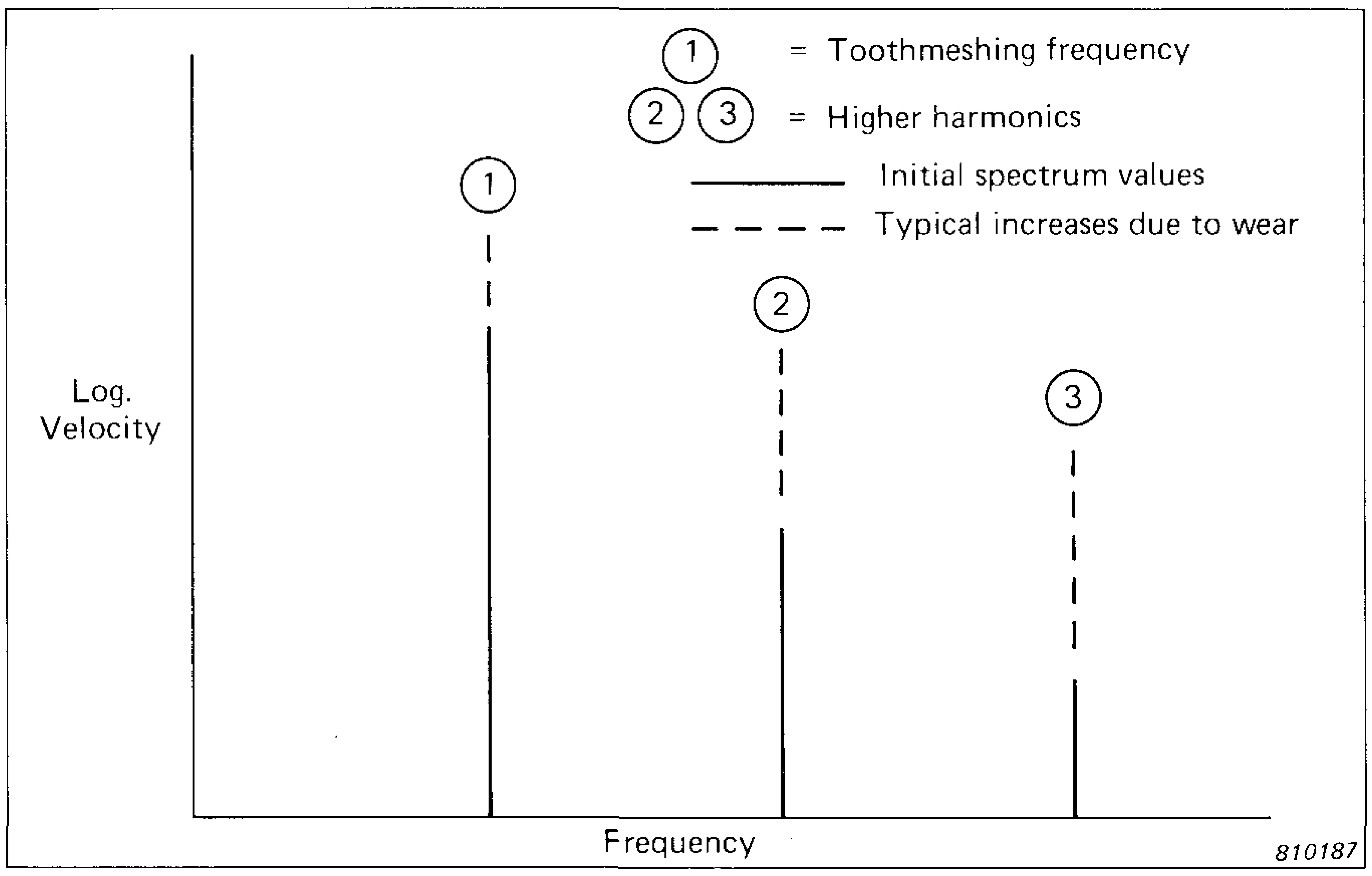

Fig.A2. Typical vibration spectrum changes due to wear

Ghost Components

As mentioned, these arise from errors in the teeth on the index wheel driving the table on which the workpiece is mounted when the gear is being machined. The frequency later generated by the gear in service corresponds to this number of teeth and therefore must be an integer harmonic of the gear rotational speed. This provides one indication that an unknown frequency component may be a ghost compo-

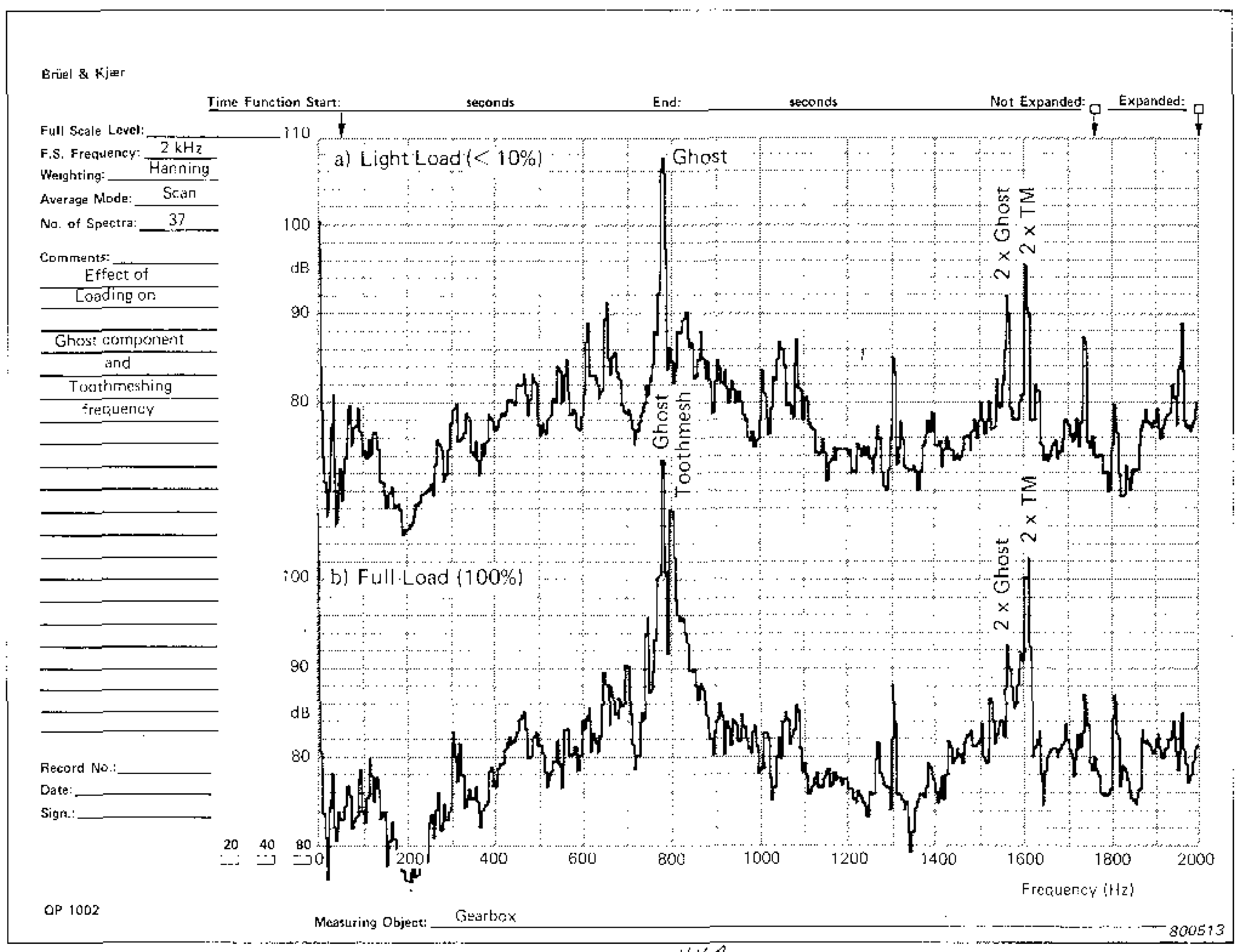

Fig.A3. Effect of loading on ghost component and toothmeshing harmonics.
(a) Light load (< 10%)
(b) Full load (100%)

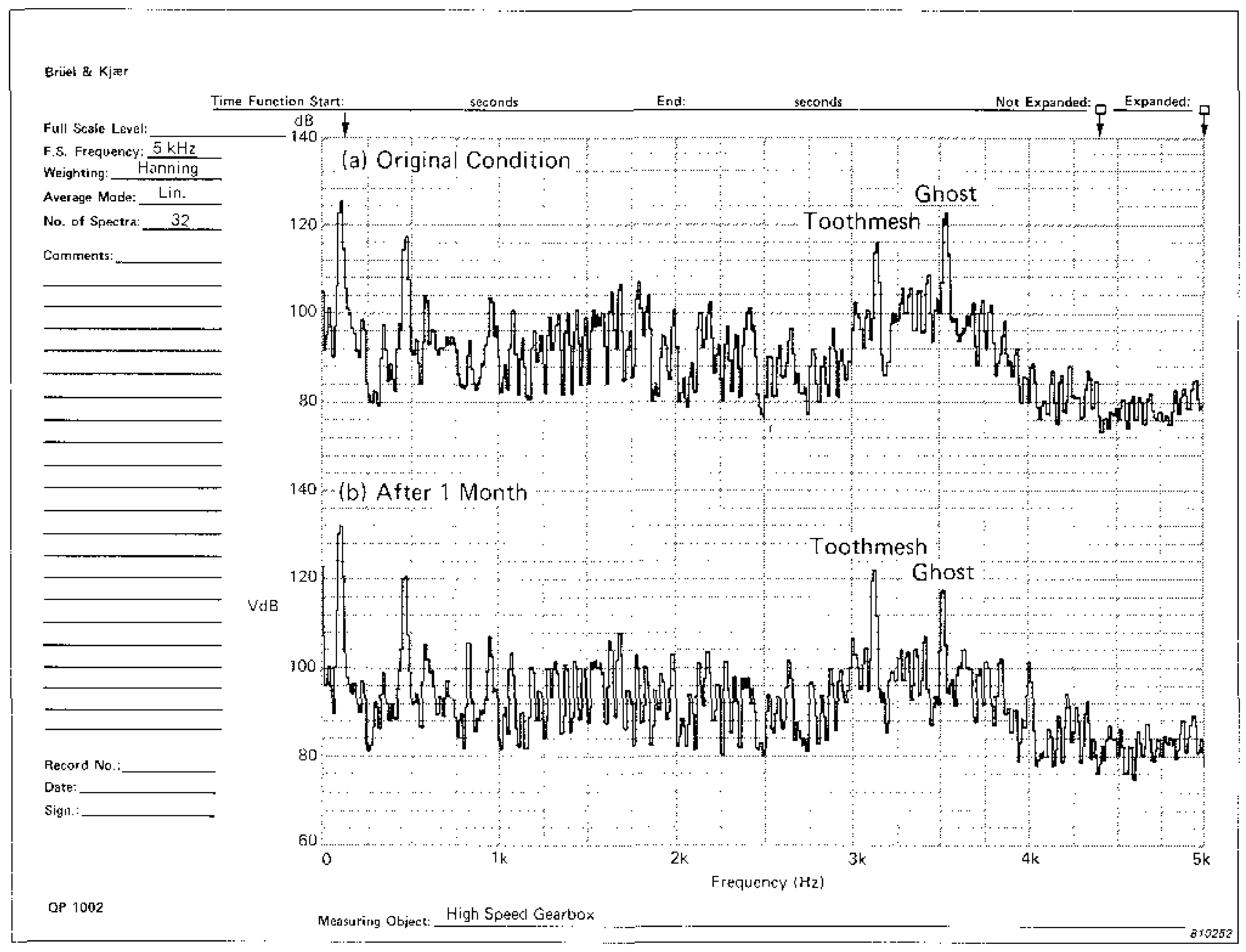

Fig.A4. Effect of wear on ghost component and toothmeshing component

nent when it is not possible to trace back to the manufacturing setup.

Another indication can be obtained from its load dependence; since the ghost effect represents a fixed geometrical error, it should not be very much influenced by load. Fig.A3 illustrates a typical case where in going from < 10% to full load, a toothmesh component (185 teeth) has increased by 21 dB while a ghost component (180 teeth) has only changed by 6 dB. The corresponding changes in the second harmonics were 7 dB and 0 dB respectively.

Once recognized, ghost components do not usually cause any problems, and if anything there is a tendency for them to become smaller with time (and wear). Fig.A4 is a typical example where over a month in which a high speed gearbox was subjected to high dynamic loading, a ghost component has gone down by about 5 dB at the same time as the actual toothmeshing component has increased by the same amount.

Modulation Effects

Components at other frequencies, in particular sidebands around the toothmeshing harmonics, can usually be explained by modulation of the otherwise uniform toothmeshing vibration. As an example, because of the load dependence of the tooth deflection effect, any fluctuations in the tooth loading (for example caused by misalignment) would tend to cause the vibration amplitude to vary in sympathy, thus giving an amplitude modulation.

At the same time these fluctuations in tooth loading must give angular velocity fluctuations and result in frequency modulation. Both amplitude and frequency modulation at a certain frequency give rise to sidebands spaced around the basic frequency (and its harmonics if it is distorted) with a spacing equal to the modulating frequency, and thus this sideband spacing contains very valuable diagnostic information as to the source of a modulation effect, often tracing it to a particular wheel in a complex gearbox.

Fig.A5 makes use of a graphical method (developed in Ref.10) to determine the influence in the frequency spectrum of an amplitude modulation effect in the time domain. Use is made of the convolution theorem, plus the fact that convolution of a delta function with another function consists in replacing the delta function by the convolving function, weighted according to the "area" of the delta function. Fig.A5(a) represents the effect of an idealised local fault on a gear, where the toothmeshing signal is assumed to be modulated by a short pulse of length corresponding to the tooth spacing (plus DC component). The pulse is of course repeated periodically for each revolution of the wheel. The spectrum of this periodic pulse consists of all harmonics of the gear rotational speed up to a first zero at the toothmeshing frequency, and the result of convolving it with all of the toothmeshing harmonics is a very flat spectrum containing a large number of low level sidebands over a wide frequency range. Normally in practice the low harmonics would also be filled in by the effects of an additive impulse (see later).

Fig.A5(b) illustrates the effect of an extension of the fault in the time signal. The broader the envelope of the fault in the time domain, the narrower and higher the envelope of the sidebands will be in the frequency domain, so that they will more obviously appear as sidebands around the toothmeshing harmonics.

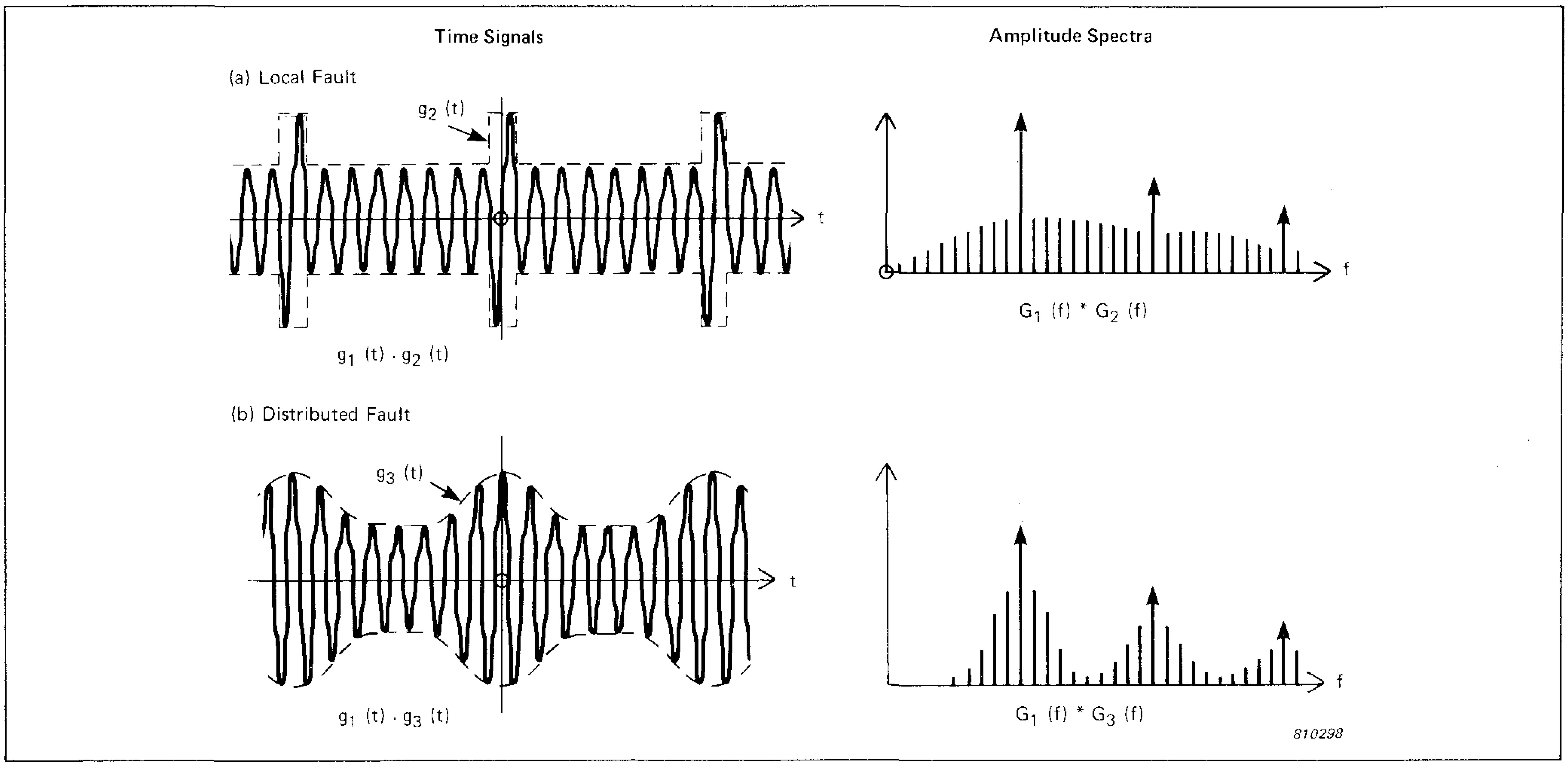

Fig.A5. Effect of fault distribution on sideband pattern
(a) Local fault
(b) Distributed fault

Even though amplitude spectra have been shown instead of the complex spectra which are strictly applicable, they represent the "average" case (Ref.10). Individual cases would only differ in small details, by reinforcement or partial cancellation of some sidebands, and the same general trends would apply. For example, the results of Fig.A5(a) agree very well qualitatively with both theoretically derived and measured results published in Ref.11 for pitchline pitting on one tooth.

Figs.A5(a) and (b) consider only amplitude modulation, and the frequency modulation which is always present would tend to modify the results somewhat further. Even modulation by a pure frequency tends to give a family of sidebands, but provided the modulation is not very pronounced, it can be represented by two or three pairs of sidebands. Thus the additional effect of frequency modulation is to increase the number of sidebands somewhat and to make the sideband patterns unsymmetrical by reinforcement/cancellation because of the different phase relationships of the sidebands due to amplitude and frequency modulation.

Additive Impulses

Both amplitude and frequency modulation tend to give signals which are symmetrical about the zero line. Any asymmetry can be interpreted as an additive impulse (Fig.A6) which is repeated once-per-revolution of the gearwheel in question and which thus gives a number of harmonics of this frequency. The amount that these extend up in frequency depends on the length of the impulse in the time signal, and for example a length corresponding to one tooth spacing would give a spectrum falling to zero by the toothmeshing frequency. Thus the harmonics below half the toothmeshing frequency are most likely due to additive effects, while those around and above the toothmeshing frequency are most likely generated as sidebands rather than as a distortion of the fundamental frequency.

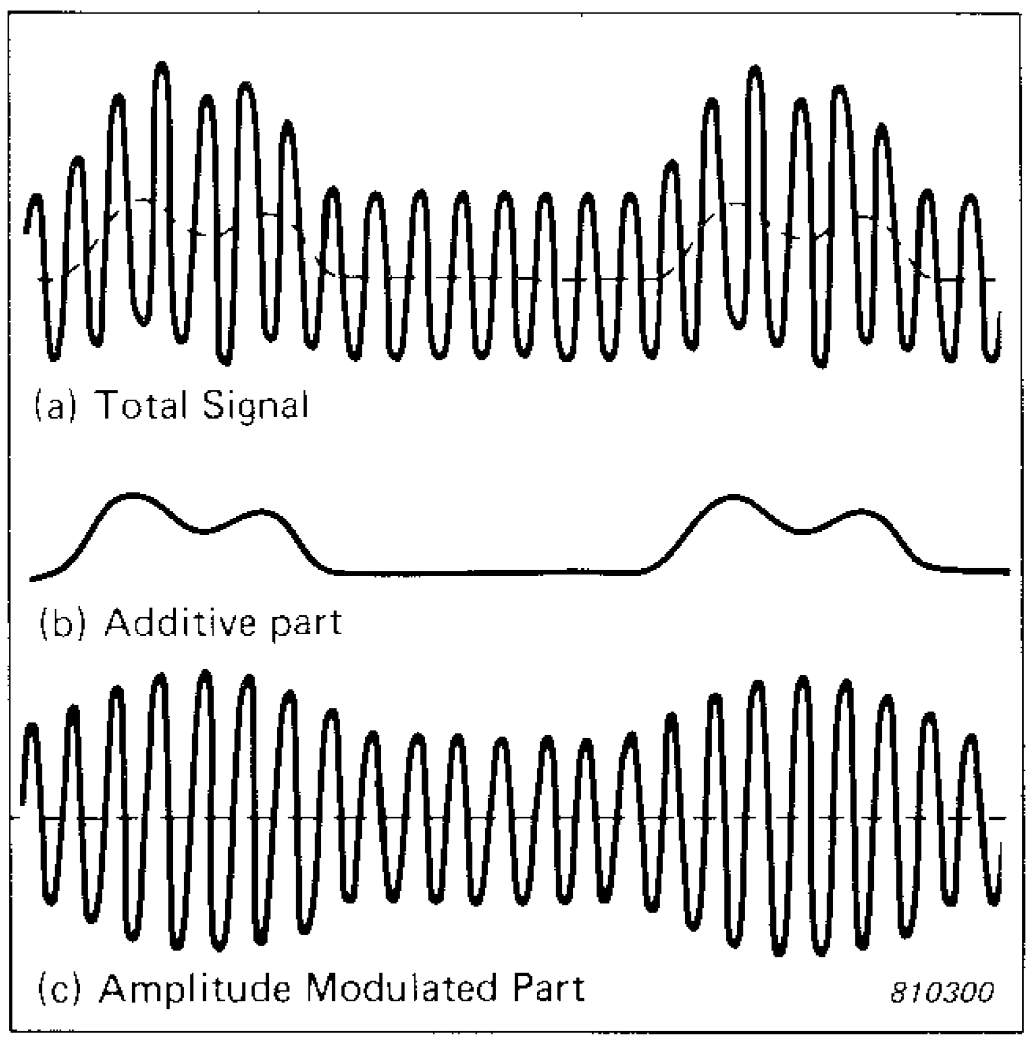

Fig.A6. Division of a gearbox signal into additive and amplitude modulated components

Intermodulation Components

Any other components in gearbox vibration spectra can usually be explained as sum and difference frequencies generated by intermodulation of the other more basic components. Once recognized, such components do not usually give grounds for concern, as they would normally only change as a result of changes in the more fundamental components, which can be related back to the physical condition as previously discussed.

Example

Fig.A7 illustrates a number of the points made in this Appendix by showing spectra from the same gearbox before and after repair (see also Figs.10 and 11 of the main text).

Taking first the spectrum after repair (Fig.A7(a)) this can be considered to represent as-new condition. It is found that the spectrum is dominated entirely by the toothmeshing frequency (333 Hz) and a ghost component 96 Hz higher. Up to the fourth harmonic of toothmesh can be located, the first two being quite prominent. The other significant component in the spectrum is an intermodulation sideband with the same 96 Hz spacing from the second toothmesh harmonic as that of the ghost frequency from the fundamental toothmeshing frequency. Some sidebands are present but at a relatively low level.

Considering next the spectrum before repair (Fig.A7(b)), firstly the overall spectrum levels are higher, particularly towards the higher harmonics of toothmesh, indicating wear (c.f. Fig.A2). Another indication of wear is that the ghost component can no longer be seen, although it is probably present at a lower level.

Finally, the large number of sidebands with both 8,3 Hz and 25 Hz spacings indicate considerable modulation by these two frequencies, as discussed in conjunction with Fig.11 of the main text. The wide extent of these sidebands, plus the fact that many of the toothmesh harmonics are appreciably lower than the adjacent sidebands, indicate a considerable influence of frequency modulation in the generation of the sidebands.

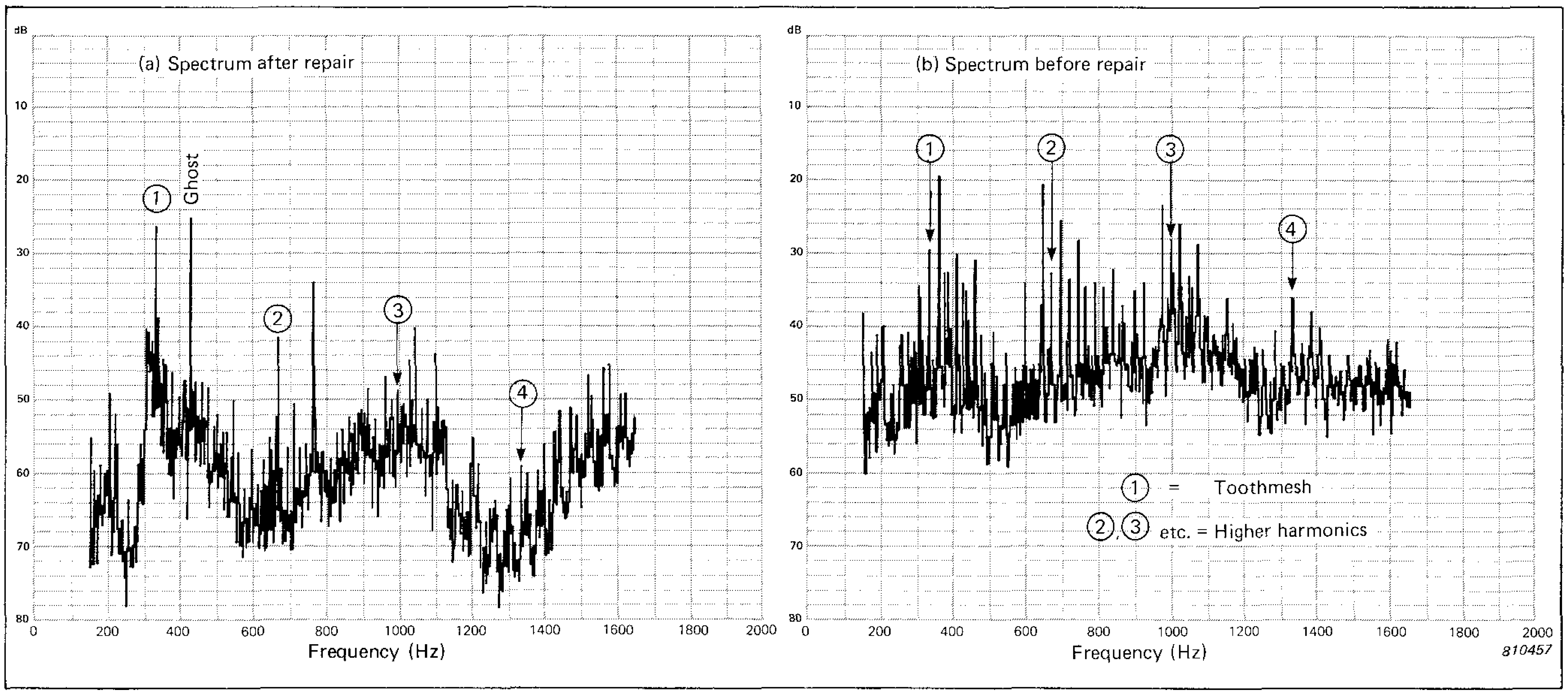

Fig.A7. Spectra from a gearbox vibration signal
(a) After repair
(b) Before repair

Appendix B — Theoretical Background

Some of the statements in the main section of the Application Note are based on theoretical aspects of Fourier analysis which will be expounded in a little more detail here.

The Fourier transform will be represented by a curved double-ended arrow with forward transformation from left to right and vice versa.

Thus: $g(t) \rightleftharpoons G(f)$

means that $G(f) = \mathscr{F}\{g(t)\}$

and $g(t) = \mathscr{F}^{-1}\{G(f)\}$

The following properties of the Fourier transform will be assumed (e.g. Ref.12):

$$g(t) \rightleftharpoons G(f) \rightleftharpoons g(-t) \rightleftharpoons G(-f) \rightleftharpoons g(t) \quad \textbf{(B1)}$$

and the relationships between the even and odd components of the real and imaginary parts of frequency and/or time functions are as follows:

$$\left.\begin{array}{lcl} \text{Real even} & \rightleftharpoons & \text{Real even} \\ \text{Real odd} & \rightleftharpoons & \text{Imag. odd} \\ \text{Imag. even} & \rightleftharpoons & \text{Imag. even} \\ \text{Imag. odd} & \rightleftharpoons & \text{Real odd} \end{array}\right. \quad \text{(B2)}$$

Since any real function can be expressed as a sum of even and odd components it follows that the spectrum of a real function is "conjugate even", i.e.

$$G(-f) = G^*(f) \qquad \text{(B3)}$$

It also follows that there is a definite relationship (Hilbert Transform) between the real and imaginary parts of the Fourier spectrum of a "causal function" (i.e. one equal to zero for negative time). Because of its relevance, a short derivation will be given here.

Fig.B1 shows a hypothetical causal function, and the way in which it can be divided into even and odd components. As will be seen, these must be identical for positive time in order that they will cancel for negative time. Thus the even component (which determines the real parts of the transform) is not independent of the odd component (which determines the imaginary parts) but in fact related by the sign function,

$$\text{i.e. } o(t) = \text{sgn}(t) \cdot e(t) \qquad \text{(B4)}$$

but $g(t) = e(t) + o(t)$

$$G(f) = E(f) + i\,O(f)$$

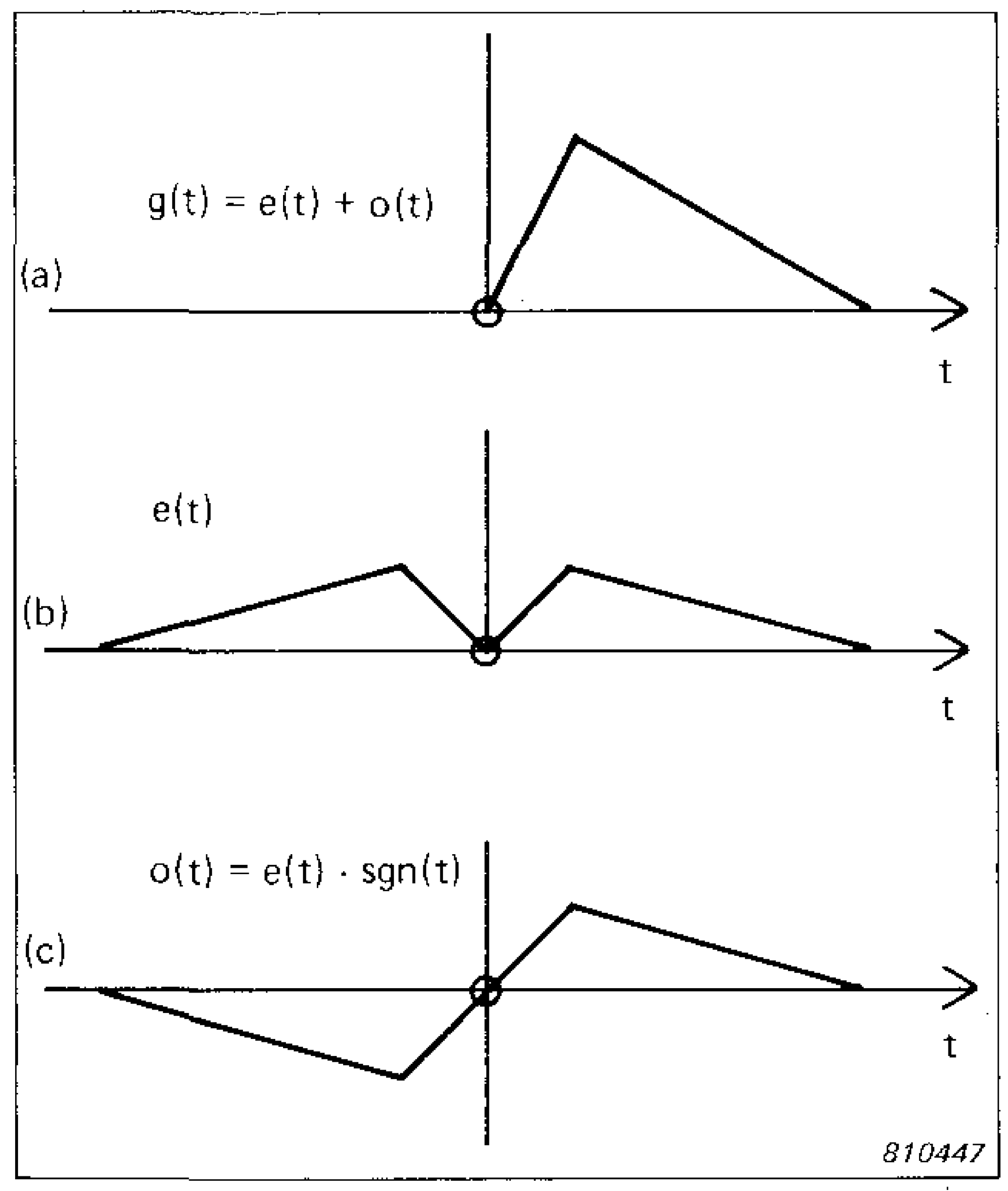

Fig.B1. Division of a causal function into even and odd components

where

$g(t) \rightleftharpoons G(f)$, $e(t) \rightleftharpoons E(f)$, $o(t) \rightleftharpoons iO(f)$

Now,

$$O(f) = \frac{1}{i} \cdot \mathscr{F}\{o(t)\}$$

$$= \frac{1}{i} \cdot \mathscr{F}\{e(t) \cdot \text{sgn}(t)\}$$

$$= \frac{1}{i} \cdot \mathscr{F}\{e(t)\} * \mathscr{F}\{\text{sgn}(t)\}$$

by the convolution theorem

$$= -\frac{1}{\pi}\left[E(f) * \left(\frac{1}{f}\right)\right] \qquad \text{(B5)}$$

$$\left(\text{since sgn } (t) \rightleftharpoons \frac{1}{i\pi f}\right)$$

which is the (inverse) Hilbert transform of E(f).

This basic information can now be used to determine the most efficient way of calculating the cepstrum.

Since the power spectrum is a real even function it follows that the logarithmic power spectrum is also a real even function.

Letting

$$C(\tau) = \mathscr{F}^{-1}\{\log F_{xx}\}$$

and

$$C^1(\tau) = \mathscr{F}\{\log F_{xx}\}$$

then from (B1)

$$C^1(\tau) = C(-\tau)$$

and from (B2)

$$C(-\tau) = C(\tau)$$

$$\therefore C^1(\tau) = C(\tau) \qquad \text{(B6)}$$

Thus the same result is obtained by a forward or inverse transform (except for a possible scaling factor, depending on the definition of the Fourier transform).

In a normal FFT analyzer an inverse transform, if available, usually assumes complex data, while a forward transform often assumes real data (this is the case, for example, with the B & K Analyzers Types 2031 and 2033). Thus, because the log power spectrum is real, it can be more efficiently transformed by a forward transform since only the half buffer size is required, or conversely, for a given buffer size the obtainable resolution is twice as fine.

Fig.B2 shows a typical example. Fig.B2(a) represents the input to a forward transform of 1024 data points in the Analyzer Type 2033. It represents a 400-line log power spectrum measured previously on the analyzer, and transferred digitally into a desktop calculator. These data values, after appropriate format conversion, were placed in samples numbers 1 to 400 of the time record. (Sample no. zero, the DC component, was set equal to the dB value of sample 1). Because the log power spectrum is in reality a real even function, the calculator program has also placed the mirror image of the 400-line spectrum (representing the negative frequency components) in samples numbers 624 through 1023 of the time function (remembering that any spectrum calculated by the FFT process is periodic).

The calculator program then caused the analyzer to perform a forward Fourier transform of this data record, treated as a time signal, and stop after obtaining the complex spectrum. Fig.B2(b) represents the real parts of the transform and Fig.B2(c) the imaginary parts. As would be expected, the imaginary parts all equal zero because the cepstrum under these conditions is a real even function.

Fig.B2(d) shows the result of allowing the analyzer to continue its process of squaring and adding the real and imaginary parts to obtain the power spectrum and then extracting the square root to obtain

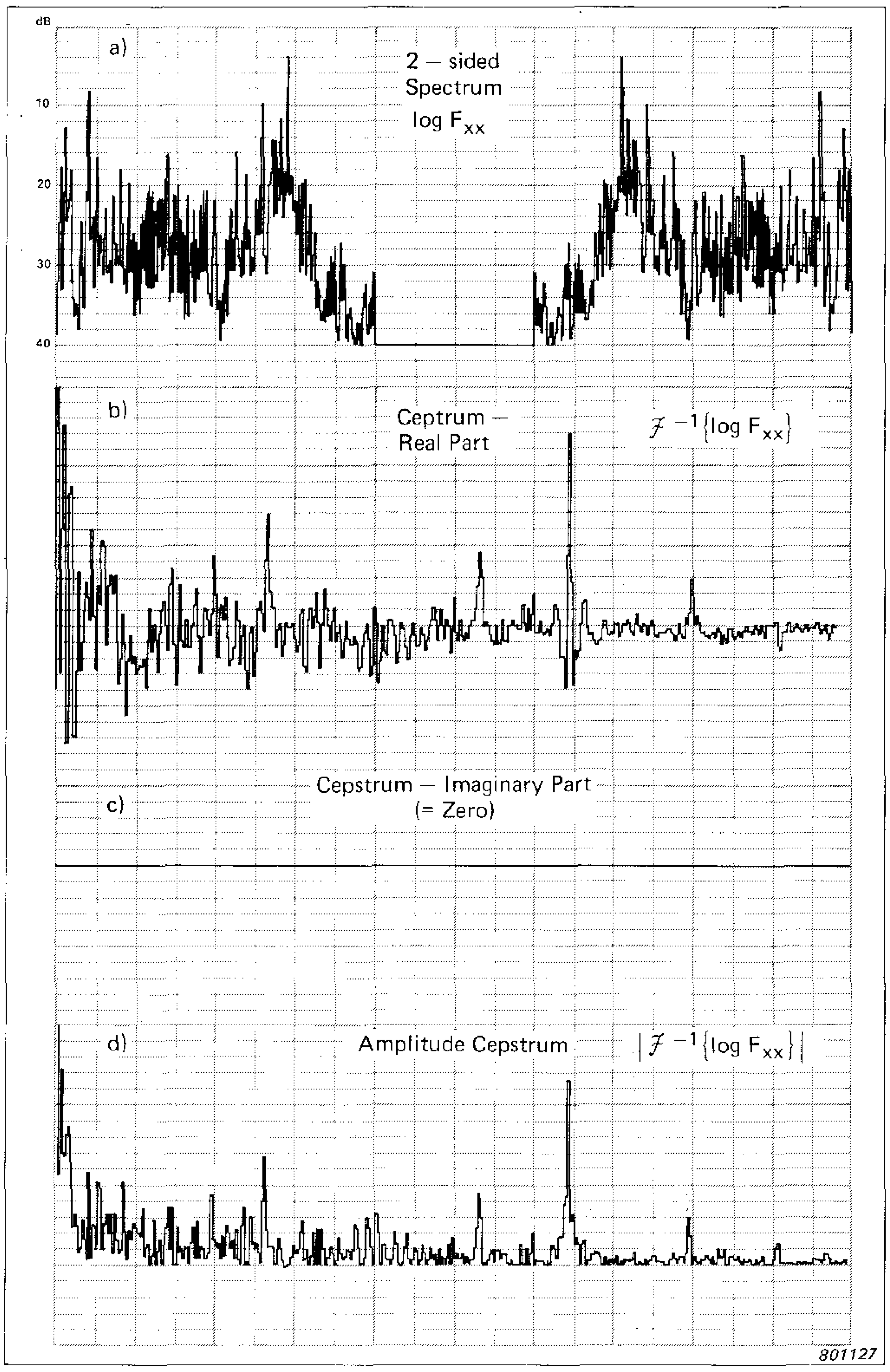

Fig.B2. Cepstrum calculation procedure — 2-sided spectrum

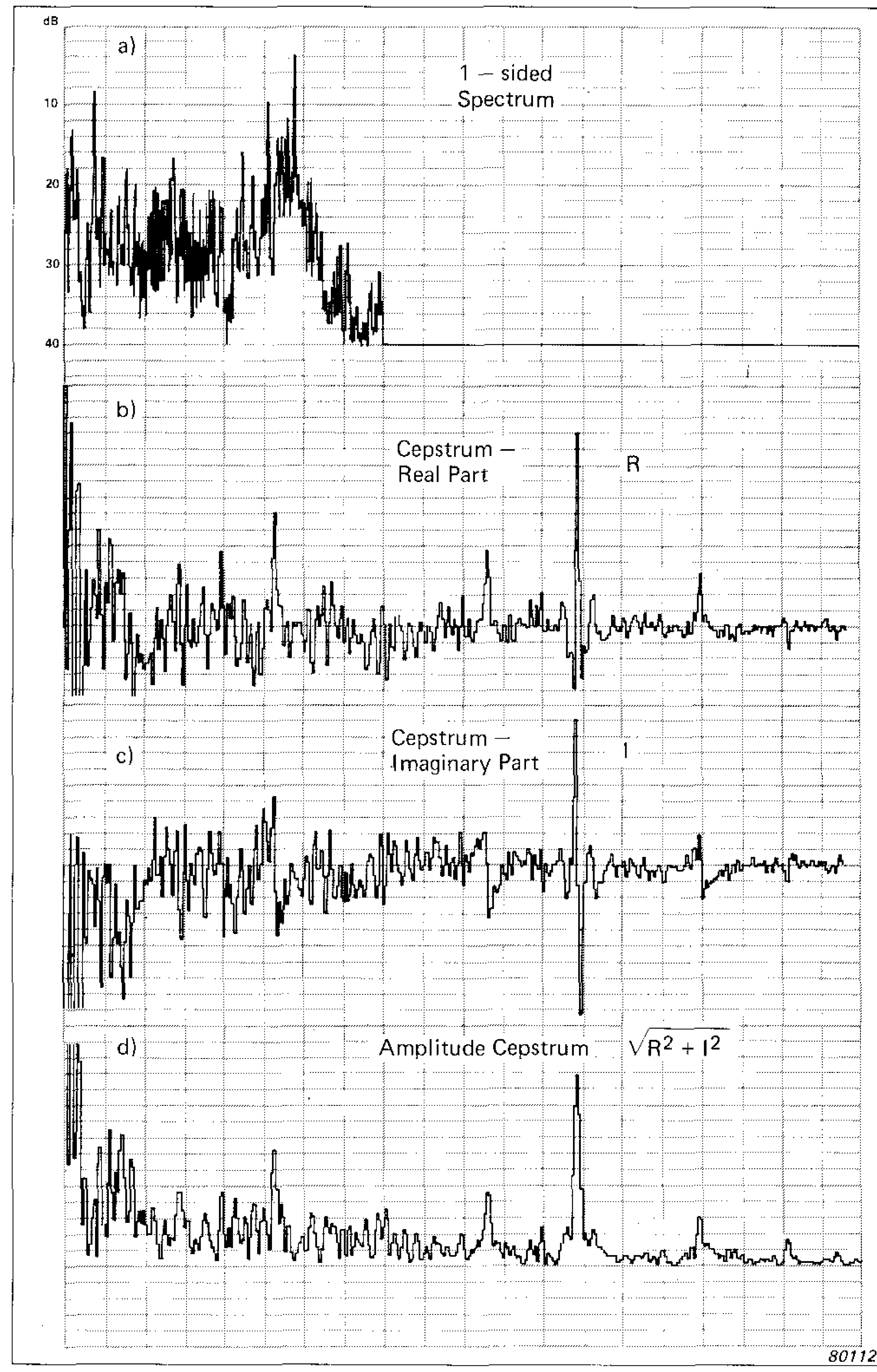

Fig.B3. Cepstrum calculation procedure — 1-sided spectrum

the amplitude spectrum. This, when displayed on a linear amplitude scale, is simply the modulus of the real part (Fig.B2(b)). It will be seen that this version is a little easier to interpret, but because phase information has been lost it is not possible to transform back to the frequency domain. The original definition of the cepstrum (Eqn.(1)) represents simply the squares of these amplitude values.

Note that the same procedure could be carried out using the Analyzer Type 2031, with the exception of the linear amplitude display of Fig.B2(d), and then the log-lin conversion could easily be done in the calculator.

The initial cepstrum software developed by Brüel & Kjær for the FFT Analyzers together with a desktop calculator used the above procedure, but it was soon realised that the insertion of the negative frequency components was not necessary. Fig.B3, similar to Fig.B2, shows the result of transforming the one-sided spectrum, leaving the negative frequency values equal to zero (and halving the zero frequency component).

As will be seen, exactly the same result is obtained for the real part of the transform (except for a scaling factor of 2 with which the result was multiplied before display). This is because it is determined by the even part of the spectrum which, by analogy with the causal function of Fig.B1, is exactly the true spectrum scaled to half size. Thus, if the cepstrum of the 2-sided spectrum is desired, the imaginary part can simply be discarded. On the other hand it will be seen that the imaginary part represents the Hilbert transform of the real part and thus has very interesting properties. For example, it will be noticed that corresponding to the peaks in the real part, there are zero crossings in the imaginary part, but a positive and negative peak on either side. It is as though the real and imaginary parts represent the projections on two perpendicular planes of a rotating vector whose amplitude gets suddenly larger in that quefrency region. In the general case the actual peak amplitude will not necessarily occur in either the real or imaginary plane, and there is a special reason why it has happened in this case.

The original spectrum of Figs.B2 and B3 is an actually measured one (from the same gearbox as Fig.2) where the sideband frequencies all coincide with harmonics of the basic shaft speed, and thus the family of sidebands includes zero frequency. As illustrated in Fig.B4(a) such a spectrum will have a cepstrum containing only positive real values. At the other extreme, if the family of sidebands does not pass through zero frequency, but is symmetrically spaced about it (such as a 2-sided spectrum of odd harmonics) the cepstrum components will be an *alternating* series with the first one negative (Fig.B4(b)). If the "phasing" of the sidebands is something intermediate, the cepstrum components will no longer be real, but complex, and the only way to determine the maximum amplitude (and the corresponding quefrency) is to combine the real and imaginary components by squaring, adding and square root extraction, which is exactly what the analyzer does anyway when it forms an "instantaneous" amplitude spectrum from the complex Fourier transform.

Fig.B3(d) shows the result of doing this, but in this case there is no advantage because the main peaks become slightly broader (due to the contribution of the imaginary part). On the other hand, where the original spectrum has been obtained by a zoom process (e.g. Fig.10 of the main text) this "amplitude cepstrum of the one-sided spectrum" is much easier to interpret. Another situation where this procedure is to be preferred because the sidebands do not necessarily coincide with harmonic frequencies is in the analysis of epicyclic (planetary) gears, where rotational speeds are not always ex-

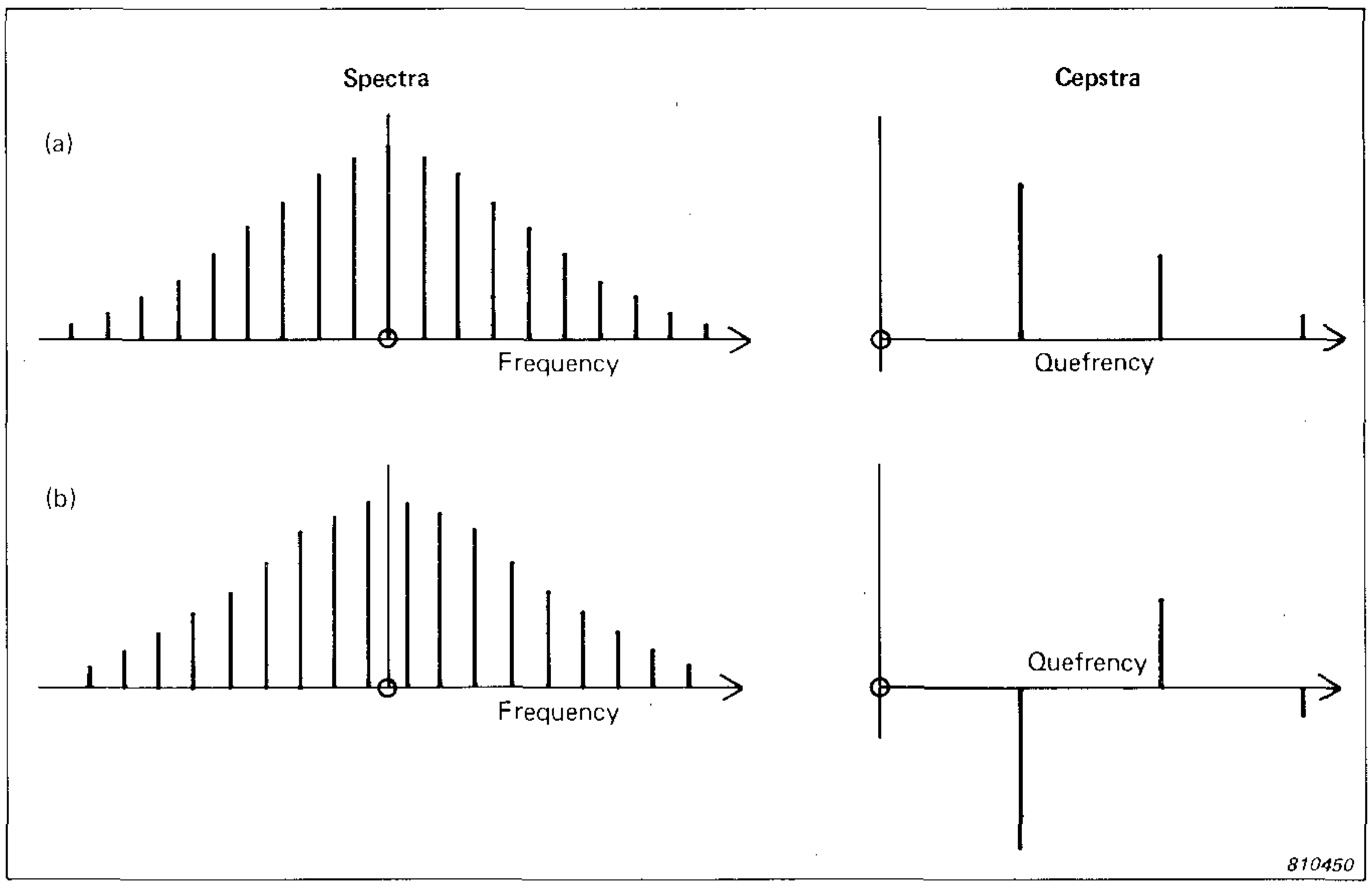

Fig.B4. **(a) Cepstrum of a harmonic series**
(b) Cepstrum of an odd harmonic series

act sub-multiples of toothmeshing frequencies.

It will be seen that there are some situations where one form of the cepstrum is to be preferred, and some where the other is better. Hence, some details will be given on how to obtain cepstra in both forms using software developed for the HP 9825 Calculator in conjunction with B & K Analyzer Types 2033 and 2031. Software is also being developed for the Tektronix 4052 Calculator and this would behave in a similar, if not identical, fashion but the discussion here is limited to the HP 9825.

BZ14 Program

The BZ14 Machine Health Monitoring program includes a cepstrum routine for diagnostic purposes. It operates on a 400-line spectrum from either Analyzer 2031 or 2033, but in the latter case the spectrum can be obtained by a "zoom" process. The "official" way of calculating the cepstrum is according to the definition of Eqn.3, but using the procedure of Fig.B3 and taking the real part only. This normally gives a satisfactory result for baseband spectra and was for example used for Figs.2, 3 and 5 of the main section.

However, it is also possible to obtain the "amplitude cepstrum of the one-sided spectrum" by a manual manipulation. When any editing of the spectrum has been completed and the actual cepstrum calculation started, the calculator, having previously *read the logarithmic spec*strum out, reads it back into the "time" memory and performs a forward transform as far as the complex spectrum (Fig.B3(b) and (c)). The analyzer microprocessor is stopped while this result is read out to the calculator, but then allowed to complete its cycle and form the instantaneous spectrum from the real and imaginary parts. This spectrum is displayed for a couple of seconds before the cepstrum result is read back as a time function, and before this happens it is possible to stop the calculator manually. It will then be found that the "time" memory contains the 1-sided log spectrum, and the "Inst." spectrum the desired amplitude cepstrum, the only problem being the scaling of the result. As regards quefrency scaling, the easiest solution is to note down the line no. of important peaks and then continue to the "official" version of the cepstrum from which one can read off the quef*rency corresponding to that line no.* (noting that where a zoomed spectrum has been transformed, the quefrency values should be multiplied by 10, as indicated by a warning message in the analyzer text line). The quefrency can in fact be read off directly from the time function before continuing on to the official cepstrum, but then the user must himself remember whether the value is to be multiplied by 10 or not.

Details of amplitude scaling will be discussed later, but at this stage it can be mentioned that the Inst. spectrum should be viewed (and possibly written out on a graphic recorder) using "Lin." amplitude scale. It will normally be necessary to increase amplitude gain until the most important peaks are optimised in height, even if this means that some very low quefrency components go over full-scale. For comparative amplitude measurements it is *valid to use the indicated mV values* since there is a fixed correspondence between these and the no. of dB they represent. Thus, to obtain comparative cepstra it is sufficient to ensure that the full-scale level in mV is the same.

To proceed to the official cepstrum it is sufficient to press "continue" on the calculator.

Zoom Cepstra

With the Analyzer Type 2033 it is possible to generate amplitude cepstra of up to 4000-line spectra (e.g. Figs.4,11) although the software so far developed for this is fairly primitive. Even so, because of its brevity, a listing of the program is reproduced here as Fig.B5, and a brief description will be given of its use.

```
0: "4000 LINE CEPSTRUM":
1: dim A$[8000+1+16]
2: buf "IN",A$,3
3: 20→C
4: ent "CENTRE LINE  OF FIRST SPECTRUM?",C
5: if C<20 or C>380;dsp "WRONG ANSWER";wait 500;jmp -1
6: 10→N
7: ent "NUMBER OF SPECTRA?",N
8: if (N-1)*40+C>380;dsp "WRONG ANSWER";wait 500;jmp -1
9: N*400→L
10: wtb 725,"#1,K0,L0,R1,O0;"
11: for I=1 to N
12: fxd 0;wtb 725,"#1,H,",str(40(I-1)+C-1),";"
13: wait 2000
14: wtb 725,"#5;"
15: tfr 725,"IN",800
16: jmp rds("IN")#-1
17: cmd 7,"_"
18: next I
19: dsp "SCALING OF SPECTRUM"
20: 0→S
21: for I=1 to L
22: S+itf(A$[2I-1,2I])→S
23: next I
24: S/L→S
25: for I=1 to L
26: fti ((itf(A$[2I-1,2I])-S)*32)→A$[2I-1,2I]
27: next I
28: 1→C
29: wtb 725,"#1,K0,L2,I0;"
30: 1025→C
31: wtb 725,"#9,"
32: for I=1 to N
33: fxd 0;wtb 725,"#1,E,",str(400(I-1)+C+511),";"
34: wtb 725,"#2,16,1,",A$[800(I-1)+1,800I];cmd 7,"?"
35: next I
36: fxd 0;wtb 725,"#1,E,",str(C+511),";"
37: wtb 725,"#1,H19;"
38: dsp
39: end
*18872
```

810050

Fig.B5. Program listing for obtaining cepstrum of up to 4000-line spectrum

The program assumes that the 10K memory of the 2033 has been filled with an appropriate time signal (i.e. correct selection of full-scale frequency so that the minimum sideband spacing to be detected is about 8 lines). It starts by requesting the "centreline of the first spectrum" which should be the line number (in the baseband spectrum) around which the lowest frequency zoom spectrum is to be zoomed. The program then requests the "No. of spectra" and this is the number of contiguous 400-line zoom spectra to be analyzed in total. For the maximum 4000-line spectrum, the answers would be "20" and "10" respectively. After the requested analyses have been performed and stored in the calculator, a certain amount of time elapses while the format of the logarithmic spectrum values is modified so that they can be read back into the analyzer "time" buffer with appropriate scaling to allow visualization. Most of this time could be saved if the scaling-up operation were eliminated. The end result would be just as good, but the spectrum in the time buffer would be so small in amplitude that a visual inspection would not be possible. It is left to the reader to make this modification if he so desires.

Finally, a scan average analysis can be made of the spectrum in the time buffer using a Hanning weighting with 75% overlap (37 spectra). This will be the required cepstrum. There will be a perfectly uniform weighting on all parts of the spectrum, since the first 1 K samples are left blank to avoid end effects. Zoom (with "Flat" weighting) can be used to measure the line number of peaks in the cepstrum more accurately. The program has not been developed to update the text-line with the quefrency corresponding to the indicated frequency, but it can be read off from the same line number in the time buffer as that obtained in the zoom analysis.

Also here, the full-scale amplitude in millivolts can be used for comparative purposes (provided the spectra were generated with the same number of lines).

Amplitude Calibration

For the two types of cepstrum discussed herein (i.e. the inverse transform cepstrum of Eqn.(3), or the amplitude cepstrum just described) the amplitude scale is linear, but the units are dB. This is because the original input to the cepstrum calculation is a logarithmic spectrum with units of dB, and the units of an amplitude spectrum are the same as those of the time signal being transformed, e.g. Volts. Thus, provided one knows the correspondance between the dB range of the original spectrum, and the number of volts representing this range when the log spectrum is treated as a time signal, it is easy to make the conversion. However, in order to make cepstra as comparable as possible, the following calibration procedure is recommended, as it is the one adopted for the BZ14 program. Note that as remarked in the main text, it is still only valid to compare cepstra made with the same analysis parameters and with the same noise level in the spectrum.

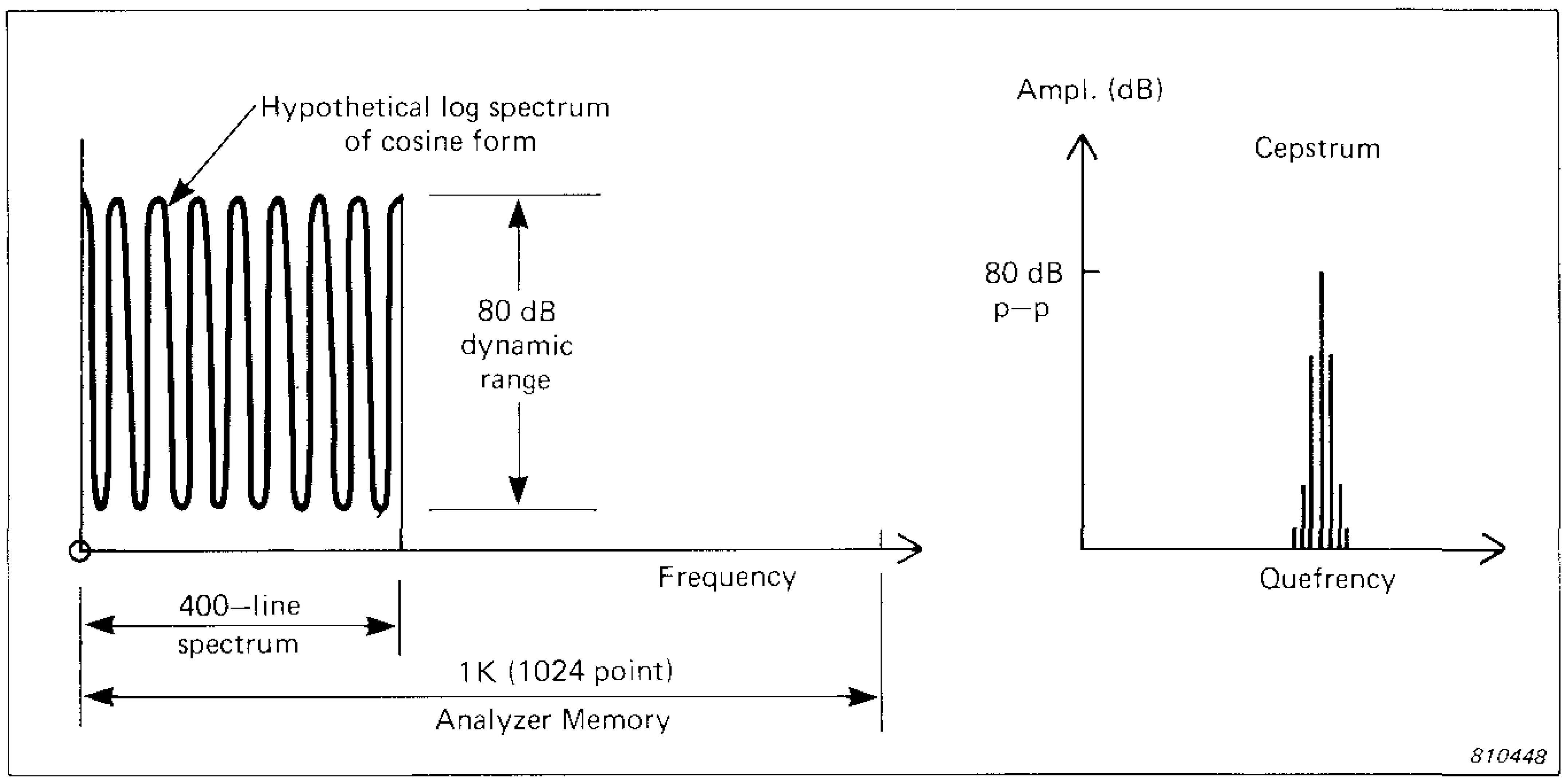

Fig.B6. Cepstrum amplitude calibration

Fig.B6 shows a hypothetical spectrum having a cosinusoidal shape with a peak-to-peak range of 80 dB over a full 400-line analysis. When it is placed into a 1024-point memory and transformed, it will be similar to a transient (tone burst) and the maximum value measured in its spectrum (i.e. the cepstrum) will be reduced by 2 factors:

(1) The measured power of the cosinusoidal component will be reduced in the ratio 400/1024.

(2) The power will "leak" into a number of lines because the bandwidth is greater than the line spacing in the ratio 1024/400.

The BZ14 compensates for both these factors and would indicate that the maximum line in the cepstrum has a value of 80 dB peak-peak as illustrated in Fig.B6. Note that BZ14 makes no compensation for any editing which may be done in the spectrum before calculating the cepstrum.

Measured voltage values in the "amplitude cepstrum of the one-sided spectrum" can be converted to the equivalent no. of dB peak-peak using the following formulae:

$$dB_{p-p} = X \cdot F_c \cdot 2\sqrt{2} \cdot \frac{1024}{400} \qquad (B7)$$

where

X = measured RMS value of cepstrum peak (V)

F_c = conversion factor, volts to dB (dB/V) $= \dfrac{81{,}92}{V_{f.s.}}$

$V_{f.s.}$ = full-scale voltage setting for the time signal representing the spectrum (typically 2,83 V)

Using the usual value $V_{f.s.}$ = 2,83 V Eqn.(B7) becomes:

$$dB_{p-p} = 209{,}6 \cdot X \qquad (B8)$$

For the zoom cepstrum of Fig.B5, the situation is slightly different. The compensation for the measured power will still be in the ratio N/N_s where

N = equivalent transform size = 9,25 K (= 9472) for scan average with Hanning window over 37 spectra

and

N_s = No. of lines of spectrum generated i.e. the appropriate multiple of 400 between 800 and 4000.

However, the analysis bandwidth should now be greater than that of the "tone burst" being analyzed, and so no further compensation for this will be required. The final formula for dB peak-peak is then as follows:

$$dB_{p-p} = X \cdot F_c \cdot 2\sqrt{2} \sqrt{\frac{N}{N_s}} \qquad (B9)$$

$$= X \cdot 36{,}2 \cdot 2\sqrt{2} \cdot \sqrt{\frac{9472}{N_s}}$$

$$= 498 \cdot \frac{X}{\sqrt{n}}$$

where

n = number of 400-line spectra generated

It is recommended that the scan *average cepstrum be used for amplitude scaling*, even if zoom (with "Flat" weighting) is used to determine quefrencies more accurately.

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